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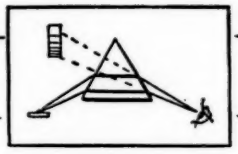

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1938

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Journal of the
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On the Reflection Factor of Clothing

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(Received June 6, 1938)

IN the modern development of illuminating engineering, a need is sometimes felt for data on the reflection factor of clothing. This is particularly true in highway lighting and motor-vehicle lighting, where a knowledge of the contrast that can be obtained between the pedestrian and his background, with various lighting conditions, is of the utmost importance.

In such problems the illumination is always low, and the only way in which the engineer can hope to secure reasonably good seeing conditions is by using high contrast. High contrast can be obtained in two ways: by using low illumination on the pedestrian and brightly lighting the pavement behind him, or by keeping the pavement brightness low and strongly illuminating the pedestrian. The former method is commonly used in street lighting from fixed lamps, while the latter is employed in lighting from motor-vehicle headlamps. In either case, a predetermination of seeing conditions requires a knowledge of the reflection properties of the pedestrian's clothing.[1] This paper presents data on the reflection factors for men's clothing worn in the United States.

Several hundred samples of cloth were obtained from four sources. Of these samples, 229 were analyzed, attention being confined exclusively to woolen cloth used in men's suits and overcoats. Spectrophotometric curves were obtained by use of the Hardy color analyzers at Massachusetts Institute of Technology.[2] The instruments irradiate the samples at normal incidence and measure the total reflected radiant flux. Because of the thoroughly diffusing nature of all the clothing samples, such reflection factors should correspond closely to those needed for the visual problem. The samples were glued to disks of white cardboard which were spun rapidly by a motor to eliminate the effect of weave and pattern.

Figure 1 shows three representative curves, one for a blue, one for a gray, and one for a brown cloth. The spectral reflection factor ρ_λ is seen to be fairly uniform throughout the visible region, a characteristic that holds for all the samples tested.

From each of the 229 curves were calculated three total reflection factors, corresponding to three kinds of incident radiant energy. The three

[1] Except, of course, when the brightness of the pedestrian is so low that it may be considered zero. See Moon and Hunt, "Reflection Characteristics of Road Surfaces," J.F.I. **225**, 1 (1938).

[2] A. C. Hardy, "A New Recording Spectrophotometer," J.O.S.A. **25**, 305 (1935). Without the help of such instruments, the present investigation would have been hardly feasible. We wish to thank Messrs. S. Q. Duntley and N. F. Barnes, who obtained the reflection-factor curves, and Messrs. M. H. Borg, and J. J. Renner, who were of assistance in the latter part of the work. We also wish to acknowledge the help of the Cornwall Mills, Inc., and Charles Adler and Son, who donated some of the clothing samples.

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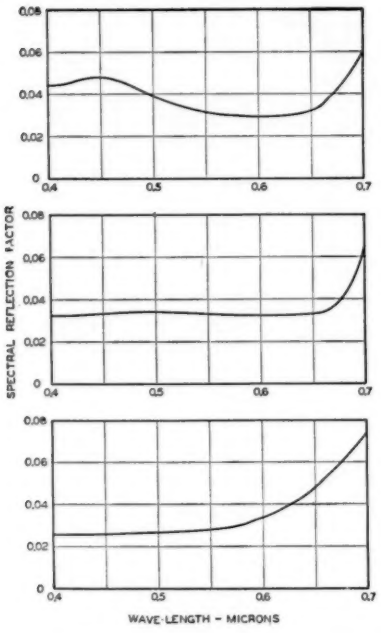

FIG. 1. Spectral reflection factor (ρ_λ) of representative clothing samples. Top, blue; center, gray; bottom, brown.

energy distributions are those obtained from important modern light sources—incandescent lamps, high pressure mercury lamps, and sodium lamps. The incandescent source was taken as a gray body operating[3] at 2842°K (standard illuminant A), and the reflection factors were obtained from the relation:

$$\rho = \frac{\int_0^\infty \rho_\lambda v J_\lambda d\lambda}{\int_0^\infty v J_\lambda d\lambda},$$

where J_λ refers to illuminant A, while v is the standard visibility function. For the mercury spectrum the energy distribution of the H-1 lamp[4] was used. The radiation from the sodium lamp was considered as homogeneous with wavelength 0.59μ.

The results for the three types of radiation were similar, though the blue samples exhibited their lowest values of ρ for sodium light, while the brown samples had their lowest ρ with mercury light. The values of reflection factor range from approximately 0.016 to 0.16, a range that seems surprisingly low. In fact, any material having a reflection factor above 0.10 would be classed as "very light" and would be selected for a suit only with misgivings about the inevitable cleaning bills. Overcoat materials, also tropical worsteds, were included in the investigation, but they were not sufficiently different in reflection factor from the suitings to justify a separate classification.

Table I gives the final results. The geometric mean for all clothing samples is 0.045 when radiation from incandescent lamps is used. The only other published data on the subject seem to be those of P. J. Bouma, as given in a paper by van de Werfhorst.[5] The Dutch results consist of a frequency curve for overcoats and another for suits, and the curves have maxima at reflection factors of 0.008 and 0.016, respectively. Evidently the clothing worn in Holland is even darker than that used in the United States.

All the data indicate that mens' clothing is predominantly neutral in tone. To study this more exactly, we selected 30 representative

TABEL I.

MATERIAL	NO. OF SAMPLES	REFLECTION FACTOR (GEOMETRIC MEAN)		
		INC.	Hg	Na
Light gray	28	0.1171	0.1164	0.1150
Medium gray	48	0.0645	0.0639	0.0624
Dark gray	23	0.0267	0.0266	0.0263
Tan	9	0.1257	0.1185	0.1271
Medium brown	23	0.0590	0.0548	0.0596
Dark brown	25	0.0302	0.0279	0.0302
Medium blue	34	0.0388	0.0382	0.0352
Dark blue	39	0.0204	0.0201	0.0195
Grays (geom. mean)	99	0.0622	0.0618	0.0607
Browns (geom. mean)	57	0.0496	0.0460	0.0499
Blues (geom. mean)	73	0.0275	0.0271	0.0257
Geom. mean of all samples	229	0.0453	0.0441	0.0439
Arithmetic mean of all samples	229	0.0550	0.0537	0.0537

[3] For $C_2 = 14{,}320$ micron degrees.

[4] Barnes and Forsythe, "Characteristics of Some New Mercury Arc Lamps," J.O.S.A. **27**, 83 (1937).

[5] G. B. van de Werfhorst, "Technical Considerations in the Lighting of Country Roads," Philips Tech. Rev. **2**, 239 (1937).

spectral reflection-factor curves and from them we calculated the trichromatic coefficients.[6] Figure 2 gives the results. The coordinates represent the usual x and y of the C.I.E. system and indicate the quality of the radiation reflected from the samples when illuminated by illuminant A. The illuminant itself is represented by the star. Though the points scatter to some extent, it should be realized that Fig. 2 includes only a small part of the color triangle and represents a comparatively small variation. The point on the extreme left represents a dominant wave-length of 0.581μ and an excitation purity of 0.46, while the point on the extreme right represents values of 0.589μ and 0.72. Over half the points are clustered closely about the star, with dominant wave-length 0.584μ and purity 0.62.

For statistical purposes, therefore, one may say that the average man in the United States wears a neutral gray suit having a reflection factor of 4.5 percent. This reflection factor is essentially the same for light from all types of lamps. The total range of reflection factors for overcoats and suits, excluding white materials, appears to be from approximately 1.6 to 16 percent.

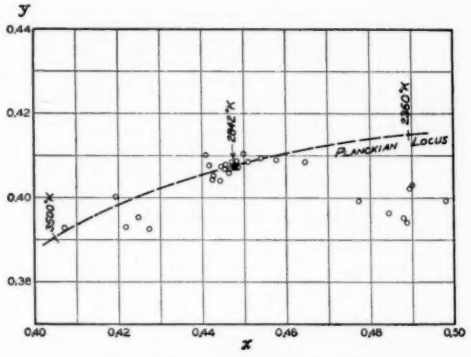

FIG. 2. Trichromatic coefficients of radiation reflected from 30 samples of clothing. Samples illuminated by standard illuminant A.

[6] A. C. Hardy, *Handbook of Colorimetry* (Cambridge, Mass., 1936). Parry Moon, *Scientific Basis of Illuminating Engineering* (New York, 1936), Chapter XIII.

Measurement of the Equivalent Luminous Intensity of Rotating Beacons[1, 2]

GERALD KENT NEELAND, MAURICE K. LAUFER AND WALLACE R. SCHAUB
National Bureau of Standards, Washington, D. C.

(Received May 3, 1938)

This paper describes a method of measuring the apparent change in the intensity of a beacon as its rate of rotation is altered. The equivalent intensity of several beacons is plotted against the duration of the flash. It is found that Blondel and Rey's law for abrupt flashes also seems to describe the equivalent intensity of service beacons to a first degree of approximation.

1. THE NATURE OF THE EQUIVALENT INTENSITY

IT is generally recognized that, as the speed of a rotating beacon is increased, the flashes of light from it not only decrease in duration but also become less visible. The reduction in visibility[3] is due to the nature of the seeing process since there is, of course, no change in the intensity distribution in the beam of the beacon.

Airway beacons give a narrow beam of light which sweeps the horizon as the beacon rotates. Let us suppose the beacon to be stopped with its beam directed toward an observer at a considerable distance. Assume now that a fixed light projector is placed near the beacon and adjusted in intensity until it appears equal in visibility to the beacon. If the beacon is now rotated at its normal speed the resulting flashes will not appear as visible as the fixed light. If, in turn, the intensity of the fixed light is sufficiently reduced, the flashes will appear more visible than the fixed light. Between these intensities of the fixed light, therefore, there is an intensity which appears equivalent in visibility to the flashes of the rotating beacon. This intensity is the "fixed-light-equivalent luminous intensity" of the rotating beacon. It will be referred to in this paper as the equivalent intensity.

2. PREVIOUS INVESTIGATIONS

Laboratory measurements of the equivalent intensity of flashing lights have been made by Blondel and Rey,[4] Piéron[5] and several others and, more recently, by Toulmin-Smith and Green.[6] Various investigators, notably Van Braam Van Vloten,[7] and Langmuir and Westendorp,[8] have attempted to apply empirical formulas, resulting from these laboratory experiments, to service beacons. It was thought worth while to test these formulas by field observations of service beacons because the formulas are based on laboratory experiments with "abrupt" flashes, having a constant intensity throughout their duration. The intensity of the flash of an ordinary rotating beacon, on the other hand, varies considerably from beginning to end. We have been unable to find a description of any measurements of the fixed light equivalents of service beacons in the literature.

3. DESCRIPTION OF BEACONS

Five beacons have been measured and their equivalent intensities determined. Three of these were of the reflecting type having reflector diameters of 24 inches, 12 inches and 8 inches, and focal lengths of 10 inches, 6 inches and 2 inches respectively; one was of the double-

[1] An investigation carried out for the Bureau of Air Commerce, of the U. S. Department of Commerce, by the National Bureau of Standards and described in detail in a report being issued by the Safety and Planning Division, Bureau of Air Commerce. Publication of this paper approved by the Director of the Bureau of Air Commerce.

[2] Publication approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

[3] The word visibility, as used in this paper, refers to the capacity of a light source to stand out from the background, i.e., the contrast between the light source and the background. Other words which have this connotation are discernability, visiopia, and vis-ability. Visibility is chosen because it is the more common synonym and hence less likely to confuse the reader.

[4] A. Blondel and J. Rey, Comptes rendus **153**, 54 (1911); **162**, 587 and 861 (1916); **178**, 276 and 1245 (1924).

[5] H. Piéron, Comptes rendus **170**, 525 and 1203 (1920); **178**, 276, 966 and 1245 (1924).

[6] A. K. Toulmin-Smith and H. N. Green, "The Fixed Light Equivalent of Flashing Lights," Illum. Eng. **26**, 304 (1933).

[7] P. Van Braam Van Vloten, L'Éclairage des Routes Aériennes, V^{e} Congrés Internationale de la Navigation Aérienne (1930), p. 5.

[8] I. Langmuir and W. F. Westendorp, "A Study of Light Signals in Aviation and Navigation," Physics **1**, 273 (1931).

ended refracting type having doublet lenses, the diameters of the outer components of which were 18 inches. The fifth beacon consisted of a 500-watt beacon lamp mounted in a beacon drum without a reflector. A diaphragm was placed on the inside of the cover glass. A circular opening in the center of this diaphragm permitted the lamp filament to be seen through an angle of rotation of approximately 30°.

A projector essentially similar to the 24-inch rotating beacon already described was used as a fixed light. This projector was mounted so that the beam was directed toward the observers. A previous calibration gave the axial candlepower at various values of lamp current.

A 500-watt beacon lamp whose horizontal candlepower, in a certain direction, was known as a function of the current, was used in most cases to determine the threshold intensity for the observers. On occasions when the atmospheric transmission was low, the fixed projector was used for this purpose.

4. Procedure

The beacon to be observed was mounted on the roof of one of the buildings of the National Bureau of Standards and the observers were stationed at one of two elevations across the city of Washington at distances of 8.3 and 2.9 miles, respectively. Telephone service made possible close coordination between observers and beacon operators.

The fixed light projector was set up about 100 feet from the beacon with which it was to be compared. The equivalent intensity of the beacon, at each angular velocity of rotation, was determined by adjusting the intensity of this fixed light until it appeared equal in visibility to the flash from the beacon.

A technique was developed in which the intensity of the fixed light was varied in steps, each of which was given a number. The observers individually recorded the number of the step at which they thought that the visibility of the fixed light was equal to that of the flash from the beacon. The intensity of the fixed light corresponding to this step number was then obtained from the records of the operator and the calibration of the fixed light. Several such determinations were made for each speed of rotation and the average intensity of the fixed light obtained for each observer. The equivalent intensity of the beacon, at any one speed and under the then-prevailing atmospheric conditions, is the average of the fixed light intensities for the several observers and is denoted by the symbol I_e.

Since this is a determination of a subjective equivalent—a psychophysical quantity—it is natural to obtain a different value for each observer. This would be true even if all observers used the same criterion for judging the equality of flash and fixed light, but it is known that different criteria were used. Some observers, for example, stated that they used the apparent size of the source as a measure of its visibility—the apparent size of both the flashing and the fixed lights, at the distance from which they were observed, being dependent on the intensity. Other observers undertook to match the fixed light with the most visible portion of the flash. Still others could give no description of their criteria. It is possible that some of the observers may have been influenced by the changes in the color temperature of the fixed light which were unavoidable with the procedure followed.

The intensity of the fixed light projector, at any given current, was known only on the axis of the beam. With this type of light source the candlepower falls off rapidly toward the edges of the beam, so that, if it were not accurately centered on the observers, values of I_e would be obtained, each of which would be larger than the true value by a constant factor.

It was decided to avoid the resulting difficulty by expressing each value of I_e in terms of its ratio to the fixed intensity of the beacon obtained in the same way, i.e., on several occasions during each test, the beacon was stopped and its beam directed toward the observing group by means of an identifying light which they displayed. The intensity of the fixed projector was then adjusted so that it equaled the beacon in visibility. If the beam of the beacon were not directed exactly toward the observers, fixed intensity values less than the true value would be obtained. Therefore, the largest of the values obtained by the group is taken as the closest approximation to the fixed intensity of the beacon and is denoted by the symbol I. Since values of both I_e and I

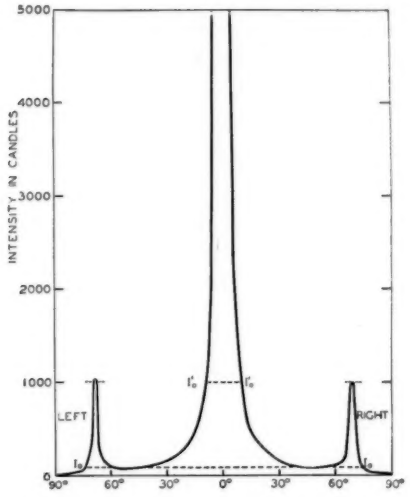

FIG. 1. Horizontal intensity distribution in the edges of the beam of the 18-inch beacon.

were determined without disturbing the alignment of the fixed projector, the ratio I_e/I is unaffected by the errors in I_e and I caused by imperfect alignment of the fixed projector because the same percentage error occurs in each quantity.

To represent the time variable we have a choice of using the duration of the flash, t, or the period of rotation of the beacon, T. There is some difficulty in determining the duration of the flash of service beacons from the intensity distribution as may be seen from consideration of Fig. 1. This figure shows the horizontal candlepower distribution of the edges of the beam of the 18-inch beacon.

On a clear night, at 8.3 miles, the distance from which the observations of this beacon were made, the intensity corresponding to the average threshold of the observers, was approximately 80 candles. This is shown in Fig. 1 by the horizontal line at I_0I_0. The length of this line indicates a flash duration of 150° or 4.17 seconds at 6 r.p.m.

There is considerable question, however, as to whether or not the relatively low intensity immediately preceding and following a peak of high intensity should be considered a part of the flash. Undoubtedly the low intensity has some effect on the sensation produced by the flash as a whole, but it is difficult to say how much. On a night when atmospheric conditions are such that the average threshold is of the order of, let us say, 1000 candles, the duration of the flash indicated by the intensity distribution is obviously shorter and probably the effective duration is somewhat shorter.

With the ordinary rotating beacon, seen through an unvarying atmosphere, t is directly proportional to T. It was, therefore, decided to plot I_e/I as a function of the logarithm of T which can be easily measured (see Figs. 2 and 3).

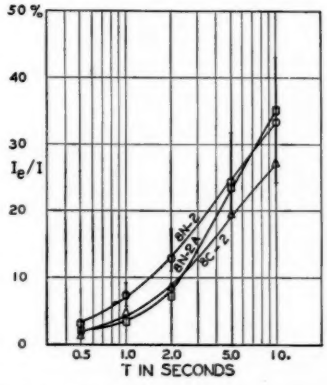

FIG. 2. Equivalent-intensity ratio in percent as a function of the period of rotation of the 24-inch beacon.

The logarithmic scale results in a more uniform spacing of the experimental values and facilitates the determination of the shape of the curve.

5. EXPERIMENTAL RESULTS

Tables I and II give the data for the 24-inch and 18-inch beacons, respectively. Some of these data are represented by the curves of Figs. 2 and 3. There is a tendency for values of I_e/I to decrease with increasing haziness of the atmosphere, the latter change being evidenced by increased threshold intensities, although some of the data seem to contradict this.

A determination of equivalent-intensity ratios for the 12-inch and 8-inch beacons was made at a distance of 3 miles. The results are given in Table III. The threshold intensities on these tests were 11 and 12 candles, respectively.

In order to correlate the results for the service and experimental beacons with those for a flash of known duration, the "abrupt-flash" beacon

was measured. This beacon was rotated at periods of 0.5, 2 and 5 seconds, giving flash durations of 0.042, 0.17 and 0.42 second, respectively. These data are also given in Table III. The intensity of the "beam" was about 1000 candles. This intensity was considerably below that for the service beacons but this was partially offset by the fact that the "abrupt flash" was observed from a distance of 3 miles, while the 24-inch and 18-inch service beacons were observed from a distance of 8 miles.

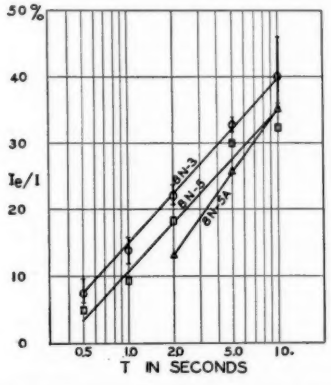

FIG. 3. Equivalent-intensity ratio in percent as a function of the period of rotation of the 18-inch beacon.

It will be noticed from Table II, that in test 8N-4 equivalent-intensity ratios are presented for the 18-inch beacon first with one face covered, and second (note "d") with neither face covered. The effect of uncovering the second face is to double the number of flashes per minute without changing their duration. The difference between the equivalent-intensity ratios for the two flash

TABLE I. *Data for the 24-inch beacon.*

Test Number	Date	Average Threshold Intensity (Candles)	Number of Observers	Equivalent-Intensity Ratio, I_e/I, in %, at Periods (Seconds) Indicated Below				
				10	5	2	1	0.5
8C-2	11–23–36	2500	4	27.0	19.4	8.2	4.6	1.8
8N-2	4–23–37	470	5	33.3	24.2	12.8	7.3	3.2
8S-1	6– 1–37	180	1	—	—	8.7	—	—
8N-2A	5–24–37	470	4	35.0	23.5	7.3	3.5	2.0
8N-2A	"	670	4	—	19.1	—	—	—
8S-2(2)	6– 4–37	130	1	—	—	6.8	—	—
8S-3	7– 9–37	110	1	—	—	7.1	—	—
Weighted* means				31.8	21.7	9.3	5.2	2.4

* Weighted according to the number of observers

frequencies did not exceed the experimental error.

Figures 2 and 3, each showing curves made at different times, give an idea of the reproducibility of the results. The spread of the values of I_e/I, obtained by different observers, is shown, for example, by the vertical lines through the plotted points of one of the curves for the 24-inch beacon, Fig. 2, and similarly for one of the curves for the 18-inch beacon, Fig. 3.

These curves also show that the increase in the equivalent-intensity ratio with increasing period of rotation is, in general, less than the corresponding percentage increase in the period of rotation (or flash-duration). In other words, the longer flashes (above 2 seconds period) have a lower equivalent intensity per unit duration than the shorter flashes. This would indicate that the

TABLE II. *Data for the 18-inch beacon. Except as stated below, all values of I_e/I are for the (double-ended) beacon with one face covered, for a clear flash, and for an observation distance of 8 miles.*

Test Number	Date 1937	Average Threshold Intensity (Candles)	Number of Observers	Equivalent-Intensity Ratio, I_e/I, in % at Periods (Seconds) Indicated Below					Notes
				10	5	2	1	0.5	
8N-1	4–14	86	5	43.7	—	—	—	—	
8N-3	4–30	70	5	39.6	32.8	22.0	13.9	7.4	
8N-5	7– 2	68	7	32.3	29.8	18.1	9.3	5.0	
Weighted means				37.8	31.0	19.8	11.2	6.0	a
Special Cases									
8N-4	5–13	520 to 4700	6	—	—	14.4	—	4.5	b
8N-5A	7– 6	10000 to 220000	6	35.1	25.8	13.1	—	—	b
8N-1	4–14	86	5	52.4	—	—	—	—	c
3N-2	5–28	11	6	—	—	15.5	—	—	d
8N-4	5–13	520 to 4700	6	—	—	14.0	9.4	5.4	e

Notes:
a—Weighted by the number of observers.
b—Threshold varied during test between the limits indicated.
c—Red (inner) lens used, thus giving a red flash.
d—Observed from a distance of 3 miles—the threshold value corresponds to about 300 candles at 8 miles under the same conditions.
e—Both faces of beacon exposed, thus doubling the flash frequency without changing the duration of the flash.

eye responds less strongly to the latter portions of the long flashes than it does to the first portions because the first portions have reduced its sensitivity.

6. The Nature of the Equivalent-Intensity Function

Our field experiments and those made in the laboratory by others show that the equivalent-intensity ratio is a function of the duration of the

flash. It is desirable, at this point, to inquire more closely into the nature of this relation.

Blondel and Rey found the following equation to hold *at the threshold* for abrupt flashes:

$$I_e/I = t/(k+t), \qquad (1)$$

where k was assumed to be a constant. They obtained values of k ranging from 0.15 to 0.30 second, depending on the conditions of the experiment, and on the observer.

In Eq. (1), as t becomes small, in comparison with k, $t/(k+t)$ becomes nearly equal to t/k and approaches zero as a limit. On the other hand, as t becomes large, in comparison with k, $t/(k+t)$ approaches unity.

The experiments of Blondel and Rey with abrupt flashes were extended by Toulmin-Smith and Green to cover the cases where I_e is maintained constant at various levels down to the threshold, I being varied with t. In order to fit their experimental curves more closely, they proposed to introduce another parameter into Eq. (1) thus giving:

$$I_e/I = Kt/(k+t). \qquad (2)$$

By choosing $K=1.1$ and $k=0.15$ second, they made this expression fit one of their curves quite closely. By properly choosing values of K and k, Eq. (2) will approximate any of their experimental curves within the limits of experimental error. Only a slight change in K is required, but very considerable change in k must be made in order to go from one curve to another. Thus, K and k are not constants but rather functions of I_e.

Blondel and Rey regarded k as a constant, probably because they investigated only the special case where I_e is maintained at the level of the average observer's threshold.

Taking $K=1$ in Eq. (2), Hampton[9] has determined from Toulmin-Smith and Green's data an empirical relation between k and the apparent illumination E_e as follows:

$$k = (0.0255/E_e)^{0.81}. \qquad (3)$$

As long as the distance and the transmission of the atmosphere between the observer and the beacon remain constant, I_e is directly proportional to E_e. Thus k is a function of I_e. Moreover, when I_e is maintained at the threshold, k is a maximum. Therefore, according to this formula, a longer[10] flash-duration would be required, at this equivalent-intensity level, than would be required at higher levels to make I_e/I approximate 100 percent.

It is interesting to see how the experimental values of I_e/I, which we have obtained, fit Eq. (1). It is important to note, however, that the experimental method of Blondel and Rey differs from ours. In their experiments, the equivalent-intensity of the flashing light was always maintained at the threshold of the observer. When the duration of the flash, t, was changed, a compensating adjustment was made in I to maintain I_e at the threshold. In our experiments we have

[9] W. M. Hampton, "The Fixed Light Equivalent of Flashing Lights," Illum. Eng. 27, 46 (1934).

[10] For E_e near the threshold level, Toulmin-Smith and Green obtained $k=0.2$ second. Therefore, a flash-duration of about 4 seconds would be required to give an apparent-intensity ratio of 95 percent under these conditions.

TABLE III. *Relation of equivalent-intensity ratio to flash duration for various beacons.*

1	2					3	4					5				
	Equivalent-Intensity Ratio at Periods Indicated (Seconds)					Beam Spread	Nominal Duration of Flash for the Periods Indicated (Seconds)					Calculated Values of k for the Periods Indicated (Seconds)				
Type of Beacon	10	5	2	1	0.5		10	5	2	1	0.5	10	5	2	1	0.5
	(percent)					(degrees)	(seconds)					(seconds)				
24-inch Airways*	32	22	9.3	5.2	2.4	4.5	.125	.062	.025	.012	.0062	.26	.22	.24	.22	.25
18-inch Airways*	38	31	20	11	6.0	7.7	.214	.107	.043	.021	.011	.34	.24	.17	.17	.17
12-inch Experimental	4.5	3.2	1.8	1.2	1.0	.75	.021	.010	.0042	.002$_1$	.0010	.46	.30	.23	.17	.10
8-inch Experimental	30	21	13	7.5	3.5	3.0	.083	.042	.017	.008$_4$	.0042	.20	.16	.12	.10	.12
"Abrupt-Flash"		78	55		20	30		.42	.17		.042		.12	.14		.17

* The 24-inch and 18-inch beacons were observed from a distance of 8 miles. (Values given are weighted means from Tables I and II.) The other three were observed from a distance of 3 miles.

maintained I constant at certain levels and determined values of I_e as a function of t.

The equivalent-intensity data for all of the beacons studied are summarized in Table III. In section 3 of the table the relative beam spreads —the angular width of the beams at the points on the horizontal candlepower-distribution curve where the intensity is 10 percent of the maximum —are given. From these widths and the periods of rotation, values proportional to the duration of the flashes have been calculated. Such "nominal durations" are shown in section 4 for the five periods of rotation. These values allow approximate comparisons with the durations of the "abrupt-flash" beacon.

In section 5 are given values of k computed from Eq. (1). Since the values of I_e vary (with the period) over a range of about 4 to 14 fold, it is to be expected that k would also vary considerably. However, in the case of the 24-inch beacon, k is sensibly constant and for all the others, except the "abrupt flash" beacon, k is an *increasing* function of I_e, instead of the decreasing function called for by Hampton's formula. This might be explained as follows: The more gradual time rate of increase of illumination at the eye for the longer beacon flashes gives more time for the desensitization of the retina.

In order to place our experimental data on a comparable basis, I_e/I values for all five beacons have been plotted in Fig. 4 against the "nominal durations." As shown, these values fall around the curve for Eq. (1) with $k=0.2$. The deviations of the points from this curve are, of course, to be expected because of the variation in the experimental values of k.

7. Conclusions

(1) The equivalent-intensity ratio increases with increasing period, i.e., with increasing flash-duration;

(2) The increase in equivalent intensity, as the beacon is rotated more slowly, is, on the whole, less than the corresponding percentage increase in period;[11]

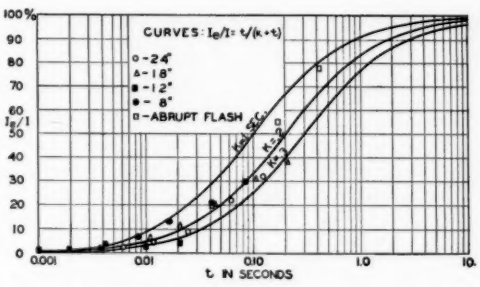

FIG. 4. Equivalent-intensity ratios in percent plotted against the nominal duration of the flash in seconds. The curves represent Blondel and Rey's law.

(3) The change in the equivalent-intensity ratio did not exceed the experimental error when the frequency of the flashes was increased without changing their duration.

(4) Hampton's formula, Eq. (3), shows k to be a decreasing function of I_e. Our data on the "abrupt flash," Table III, tend to corroborate this, but the values of k for the experimental and service beacons show that, on the whole, k is an *increasing* function of I_e.

(5) Blondel and Rey's Eq. (1), with $k=0.2$ second will give values of I_e which may be used for equivalent-intensity calculations for service beacons to a first degree of approximation.

[11] Conclusion (2) states that, in general, the equivalent intensity of a flash is not directly proportional to its luminous energy, J, but depends upon the duration, t— the shorter the duration, the greater the effect per unit of luminous energy. This result follows also from Eq. (1), for abrupt flashes. Since:

$$J \propto It,$$
$$J \propto I_e(k+t),$$

where I_e=a constant, since, in Blondel and Rey's experiments, it is held at the observer's threshold and in Toulmin-Smith and Green's experiments it is held constant at various levels. Thus, the energy required for constant visibility is a linear function of the duration of the flash, the longer flashes requiring more energy and hence being less effective per unit of visual energy. This corresponds to a lowered retinal sensitivity for the long flashes.

The Influence of Acoustic Stimuli Upon the Limits of Visual Fields for Different Colors

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(Received May 13, 1938)

IN our previous work[1] it was established that the acoustic stimuli noticeably broaden the limits of the visual field for green light.

The experiments made by Professor S. V. Kravkov[2] have shown that the sensibility of the macular cones undergoes quite different changes under the influence of sound, according to the wave-length of color stimuli. Thus, in connection with the above statement, it occurred to us to investigate the influence of the indirect acoustic stimuli upon the visual field for color stimuli of different hues. Likewise, we realized that we must not only subject to investigation the influence of sound stimuli but to experiment with noise stimuli as well. With these aims in mind, and under the guidance and initiative of Professor S. V. Kravkov we accomplished this research work.

METHOD

We measured the size of the visual field with a Hardy perimeter. This instrument is made up of a foot to which an arch with a slider is affixed; the slider moves from the center to the periphery. A small two-volt electric bulb was placed on top of the slider in a special cylinder. The electric bulb served as a source of light which acted as a stimulus. The voltage was kept constant. The color was given by placing in front of the bulb, acting as a stimulus, a glass color filter. Then the glass of the color filter was inserted into the nontransparent cover of the metal cylinder, which had a circular aperture in the center, of 1.5 mm in diameter. This color spot actually served as a color stimulus.

The perimeter was on the table in a special cabinet, open only to the side of the observer. The inside surface of the cabinet and the arch of the perimeter were pasted with white chalk paper and were illuminated by special bulbs, which threw about 30 lux upon the wall of the cabinet. The eyes of the observer were thus adapted to the above lighting conditions during the time of the experiment.

We made use of a sound generator with a cathode lamp as a source of acoustic stimuli.

The sound, we took to act as stimulus, had a loudness of approximately 75 decibels and a frequency of 780 Herz (vib./sec.). The sound was transmitted to both ears of the observer by telephone receivers.

The noise of one or two 25-watt motors of alternating current served as noise stimuli. The motors were placed to the right of the observer, at a distance of 1.5 m. Although we did not establish the loudness, produced by the noise of the motors, we may well state that it was sufficiently high.

Two observers participated in our experiment. We used the following color filters in the tests: (1) red, (2) orange-red, (3) green and (4) blue.

Each of the above was tested under three different conditions: (1) with sound, (2) with noise, (3) without acoustic stimuli.

In our experiments we used the following color filters: three Wratten color filters, No. 26 (orange-red), No. 53 (green) and No. 47 (blue), and one filter (red) that was not standardized.

The transmission of light by the color filters may be characterized as follows:

Red: transmits rays with a wave-length exceeding approximately 625 mμ; with a maximum transmission at 700 mμ.

Orange-red: transmits only rays with a wave-length exceeding approximately 580 mμ; with a maximum transmission at 680 mμ.

Green: transmits rays with a wave-length ranging from approximately 475 mμ to 625 mμ; with a maximum light transmission at 540 mμ.

Blue: transmits rays with a wave-length of approximately 535 mμ and shorter; with a maximum transmission near 440 mμ.

It took the observer from 8 to 10 minutes to adapt himself to the light condition in the cabinet where the perimeter was kept, before

[1] P. Yakovlev, *Visual Sensations and Perceptions* (Moscow 1935), p. 127 (Russian).
[2] S. Kravkov, Acta Ophthalmologica **14**, 348 (1936).

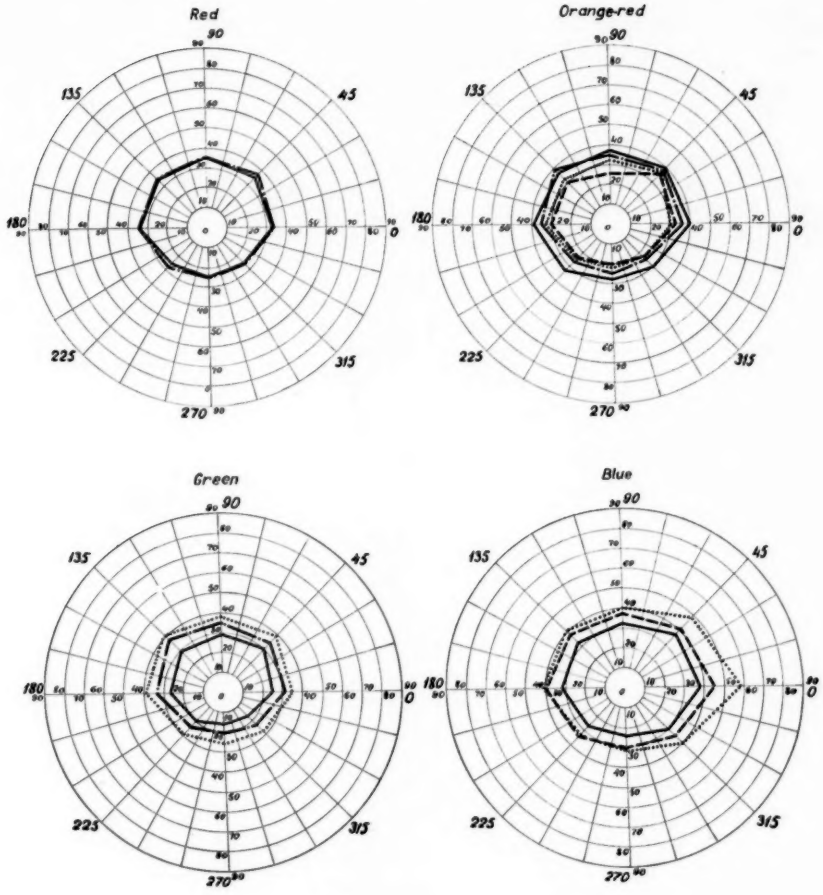

FIG. 1. Observer M. ——— without acoustic stimulation, - - - - sound, — · — noise 1, ... noise 2.

commencing his observations. Then a band was put across his left eye and his head was kept in position by a special head supporter. The observer through the entire test was concerned with one problem, namely, to keep his right eye focused upon the immobile point of fixation in the center of the perimeter and thus to observe closely the disappearance of the hue of the color spot, gradually sinking into the periphery. Hence, the observer was requested to make note not of the vanishing of the light stimulus point in general but of the disappearance of the specific color (hue) tested (green, blue or red).

At the beginning, as a rule, the specific threshold for color is defined without the interference of sound or noise; afterwards with sound or noise.

In the interval, between the first and second series of determinations which lasted from 5 to 7 minutes, the eyes of the observer were adapted to the same brightness inside the cabinet. On certain days a third control series of measurements was performed, likewise without the interference of sound stimuli. This control series was preceded by a rest period of a few minutes.

The moment when the color sensation vanished in the periphery was defined by the 8th meridians of the circumference in every 45 deg., that is,

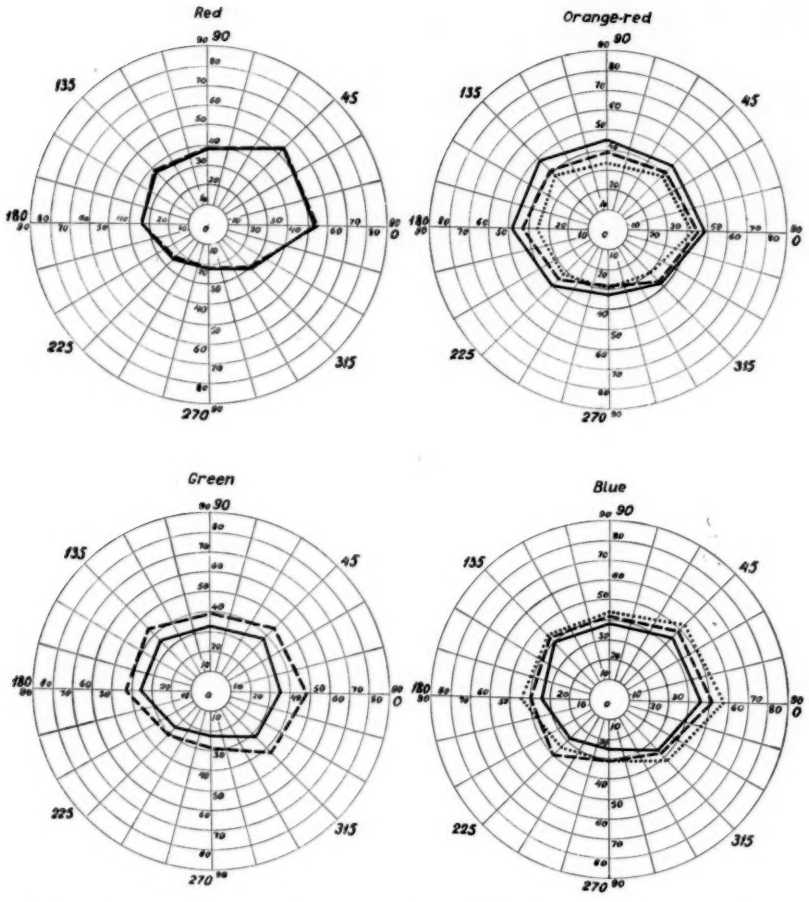

FIG. 2. Observer P. ——— without acoustic stimulation, – – – – sound, ... noise 2.

by the meridians respectively at an angle of 0°, 45°, 90°, 135°, etc.

As the blind spot lay across one of the meridians (0°), the observers were conscious of its presence and hence continued their investigations even after the color stimuli had left the zone of the blind spot.

The effects of different degrees of loudness upon the sound stimuli were put to the test in separate experiments. Thus, sometimes, besides the settings, determined under conditions of noise produced by one motor (indicated below as "noise 1"), when a second motor was turned on, another setting was established (indicated below as "noise 2").

RESULTS

The results reached in our tests are given in Figs. 1 and 2 where the fields of vision for different colors on different meridians and under various conditions (without acoustic stimuli or with one or another acoustic stimuli) are indicated.

The lines are traced on the dots, each of which is the mean value of several days experiments (on the average of three days).

An examination of the figures will clearly show that acoustic stimuli may considerably change the limits of color visual fields. The effects of the acoustic stimuli may be different,

depending upon the color for which the limits of the visual field are defined. For the green and blue colors, the limits of the visual field under the influence of acoustic stimuli expand; for the orange-red color, they, on the contrary, contract; for the red color, they remain without any change at all. These facts referring to the cone vision of the peripherical parts of retina, are in complete harmony with Professor S. V. Kravkov's discovery for macular vision.

Those cases, where the influence of acoustic stimuli of different intensity was experimented upon (see data of experiment: Observer M, with orange-red and green colors), lead us to deduce that with an increase in the intensity of the sound stimulus, its influence upon color sensitivity likewise increases, whereas the character of the changes remains the same.

Summary

1. The acoustic excitations may influence the limits of visual fields for color stimuli.

2. The limits of the visual fields for the green and blue colors expand under the influence of acoustic stimuli; for the orange-red color, they contract; for the extreme red color they do not change at all.

3. The color sensibility of the peripheral part of retina changes under the influence of acoustic stimuli very much like the change of the color sensibility of the macular regions of the retina.

An Interval Recorder for Analysis of Spectra

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(Received April 18, 1938)

A device is described which automatically combs a list of wave numbers punched on a paper tape for pairs of lines having constant differences within a given tolerance, and records the wave numbers of these pairs on motion picture film. It supplements the spectrum interval sorter previously described, which makes use of the same punched tape and automatically places all similar intervals in juxtaposition so that frequently recurring intervals can be detected. The interval recorder is set successively for the various intervals which the interval sorter chart shows to be important. When a light beam passes simultaneously through holes in both upper and lower sections of a looped portion of the tape, a detecting photo-cell flashes an argon lamp which momentarily illuminates two wave number dials turning in synchronism with the motion of the tape, and records their readings. By means of a calibrated differential the two dials and the tape can be set for any desired wave number difference. By controlling the width of the scanning beam the tolerance for recording can be set equal to the interval variation which it is desired to include. With the aid of this machine and the interval sorter some 2000 lines in the spectrum of Ce II have been classified in a quadratic array by Albertson and the writer. The average difference between observed and calculated intervals, with wavelength measurements made on the automatic computing comparator described by the writer, is slightly over 0.01 cm^{-1}.

THE mechanical spectral interval sorter previously described[1, 2] has, over a period of years, proved its value in the analysis of a number of complex spectra.[3] Experience has suggested several improvements which have been incorporated in the machine. The likelihood of breakage of the paper tape caused by its tendency to climb on the sprocket teeth has been practically eliminated by the addition of a positive drive to the second sprocket (*B* in Fig. 2 of the second reference), which is now geared to the driving sprocket through a connecting shaft. In this way the load on the tape is reduced, and catching of the upper section of the tape on its returning section, caused by excess sprocket tooth pressure on the edges of the tape perforations, is eliminated.

As pointed out in the second reference, such an interval sorter is of value for two purposes. In the first mode of use selected categories of spectrum lines are sent through the sorter, which groups all identical intervals together on a chart and makes apparent, by the frequency of occurrence of the various intervals, which are important in the energy level diagram which is being sought. In the second mode of use a complete wave number list is automatically combed for lines having a given wave number difference which is already known to be important. We have found the interval sorter satisfactory for both purposes, but somewhat clumsy for the second in that six or eight feet of photographic paper 40 inches wide must be used to assure without further subtraction positive identification of the two lines which have combined in any given interval. A second machine was therefore designed to record directly the wave numbers of all pairs of lines showing a given interval. This, which we may call a spectral interval recorder, has been found to supplement the interval sorter very effectively.

A photograph of the front of the interval recorder, with the cover removed from the dial and lamp housing, is shown in Fig. 1. Its general principle of operation is depicted in Fig. 2. The punched tape, which can be used in lengths up to 1000 feet (though 400 feet usually suffices), is unwound from a reel at *A* by the pull of the upper sprocket *S*. From this the tape passes through a loop of from eight inches to thirty feet to the lower sprocket *S'*, from which it is coiled up on a reel at *B*. A beam of light from a small lamp *L* is focused on the tape at *S*, the width of this beam being adjustable to correspond to the wave number spread which it is desired to include under a given interval.

[1] G. R. Harrison, R. S. I. **3**, 753 (1932).
[2] G. R. Harrison, R. S. I. **4**, 581 (1933).
[3] W. E. Albertson, Phys. Rev. **45**, 304 (1934), Os I; W. E. Albertson and G. R. Harrison, Phys. Rev. **52**, 1209 (1937), Ce II; unpublished material on Nd I, Nd II, Sa I, Gd II, Ce I, Ru I.

The diverging light beam, after passing through the upper tape and the sprocket S (which, like S', is cut away to allow its passage), is focused by lens L_2 on the lower tape, and then diverges again to fill photo-cell P.

When the holes in the upper and lower tape sections are in line the light beam passes through both sprockets, and when the photo-cell is illuminated the resulting current pulse, after amplification, trips a thyratron, which discharges a 12 mf. condenser charged to 1000 volts through an argon-filled U-tube which illuminates the dials. The trip circuit used is similar to that shown at D in Fig. 7 of a paper by the writer in which an automatic spectrum comparator is described.[4] The argon tube flashes for a few microseconds only, thus enabling an $f:4$ lens of 2-inch focus to produce an unblurred image of the readings of the two rapidly rotating wave number dials. The two dials are attached to the shafts carrying sprockets S and S', and since a sprocket similar to these is used on the punching device with which the wave numbers are recorded on the tape, tape and dials move in wave number synchronism. A camera then records on motion-picture positive film the wave numbers corresponding to any two holes which pass together through the light beam. The two sprocket shafts are connected by a differential which carries a dial calibrated to read the wave

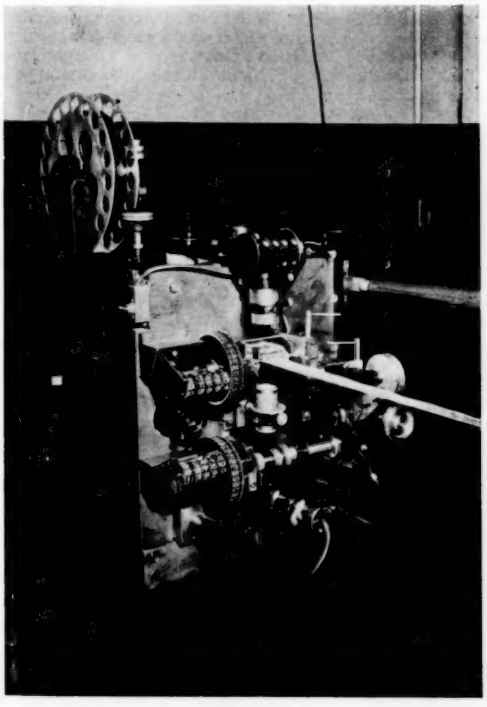

FIG. 1. The spectral interval recorder, with cover, lamp, and mirror removed from in front of the wave number dials and camera lens.

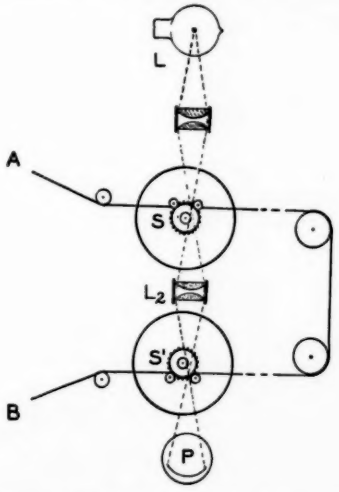

FIG. 2. Diagram showing the principle of operation of the interval recorder.

number difference between the dials of S and S'. After the tape has been threaded through the machine this differential can be set and clamped for any desired interval. A small reversible variable-speed motor drives the lower sprocket shaft directly, and the upper sprocket through the differential.

Some backlash in the gears is inevitable, and to compensate for this a lever which governs the direction of travel of the tape by controlling the drive motor is attached to a link motion which moves the lens L_2 slightly when the tape is reversed.

While an attachment designed to advance the film automatically every time an interval is recorded would be comparatively easy to construct, this has thus far not been added because of the ease with which the film can be advanced by hand with a ratchet device whenever the

[4] G. R. Harrison, J. O. S. A. **25**, 169 (1935).

light flashes. Even if two intervals occur so close together that the film cannot be advanced properly between them, the overlapping numbers can still be read.

Figure 3 shows two short sections of records obtained with the interval recorder, which has thus far been applied only to the analysis of Ce II, Fe I, and Ru I. Once the tape has been punched, a few minutes suffices to comb a list of several thousand lines for a given interval, and one tape will serve for at least 150 intervals before it wears out. As many as 150 line pairs showing the same interval within a narrow tolerance may occur in a complex spectrum, a fraction of these being, of course, the result of chance.

Setting and operation of the machine can be left in the hands of an unskilled clerk.

With the aid of this machine and the interval sorter, and with wave-lengths obtained with the automatic computing comparator, more than 2000 lines of Ce II have been classified in the past few months by W. E. Albertson and the writer.[5] Many of the differences between calculated and observed wave numbers are found to be less than 0.01 cm^{-1}, from which we find that the probability of getting by chance a quadratic array such as has been set up with the aid of the machines is less than 1 in 10^{100}.

One of the best methods for testing the accuracy of a set of wave-length measurements is by applying the Ritz combination principle to a term array. After the best values for the terms of Ce II have been found from our new wave-length measurements on the more than 7000 Ce II lines which we have observed, we expect to calculate the wave-lengths of the lines and thus improve the wave-length values for many lines. At present the precision for most lines appears to be within ±0.002A, but by using the smoothing procedure outlined it is expected that most lines can be determined to within ±0.001A, at least for wave-lengths between 4500 and 2000A. The cerium spectrum is being measured between 10,000 and 1,000A, some 12,000 lines having been recorded thus far.

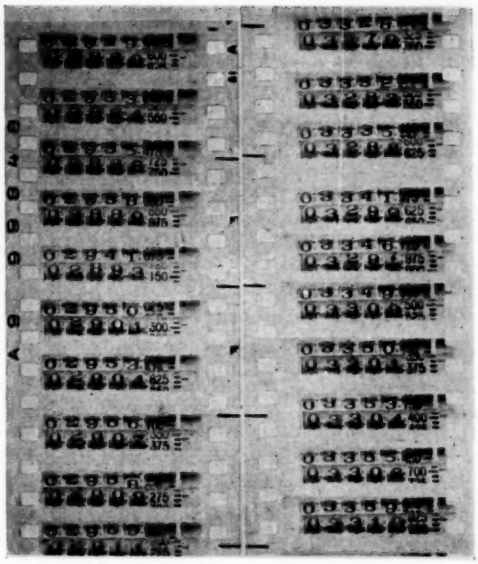

FIG. 3. Two portions of records from the interval recorder. In two mirrors which bring the readings into juxtaposition, the readings of the two wave number dials are photographed on positive film whenever an argon lamp is flashed by the passage of holes in the tapes crossing the scanning beam.

The interval recording machine is also being used to test a new list of wave-lengths of iron being prepared by Professor D. W. Weeks of Wilson College and the writer. More than 14,000 lines of Fe I and Fe II measured with the automatic comparator are being checked by means of the combination principle, and while the wave-lengths show the same consistency as the Ce II wave-lengths, the list can be smoothed somewhat further by this procedure.

[5] A preliminary report on this analysis appeared in Phys. Rev. 52, 1209 (1937).

The Correlation of Color Temperatures Based on the Wien and the Planck Radiation Formulas

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(Received January 14, 1938)

THE scientific specification of temperature colors has been developed in two different fields, that of color temperature calculation and that of colorimetric calculation. Since these methods do not in all cases yield equivalent results, the correlation presented below will be of value in bringing these methods together.

Color temperature calculations are greatly facilitated by the use of reciprocal temperature instead of temperature itself. This was proposed by Priest[1] with the thought that this reciprocal scale would be of particular value in pyrometry and colorimetry not only because computations take on simpler form, but also because on the reciprocal scale the least perceptible chromaticity difference is a constant independent of temperature and has the convenient approximate value of one micro-reciprocal-degree (abbreviation "mired").

The I. C. I. standard observer and coordinate system[2] is, of course, admirably adapted to the specification of color stimuli in general. Its usefulness for exact or approximate temperature colors is enhanced by the availability of isotemperature lines[3] by which color points slightly off the Planckian locus can be assigned an appropriate color temperature.

Unfortunately these two methods are not on the same temperature scale because, while the colorimetric calculation method is based on distributions of spectral energy in accordance with Planck's formula, the scale of reciprocal temperature develops mathematically from the Wien radiation formula as has been shown by Gage.[4] At low temperatures or short wave-lengths the Wien and Planck formulas yield the same results but at larger values of temperature or wave-length the differences become significant.

A conversion table has, therefore, been prepared by which reciprocal temperatures may be converted from the Wien to the Planck basis for use in colorimetric computations or for other purposes. The series of spectral energy distributions which form the basis for the table could be computed directly from the Wien formula but can be more conveniently obtained from Planck energy distribution data.

If the Planck and Wien formulas are expressed in the form

$$E_\lambda \propto C_1\lambda^{-5}(e^{C_2/\lambda\theta}-1)^{-1} \qquad (1)$$

and

$$E_\lambda \propto C_1\lambda^{-5}(e^{C_2/\lambda\theta})^{-1} \qquad (2)$$

respectively, it follows that

$$(E_\lambda)_w/(E_\lambda)_p=(e^{C_2/\lambda\theta}-1)/e^{C_2/\lambda\theta}. \qquad (3)$$

This ratio permits values of spectral energy by the Wien formula to be conveniently obtained from readily available tabulated data[5] based on the Planck formula. Values of the ratio in Eq. (3) have been computed over a considerable range of $\lambda\theta$ and are listed in Table I.

TABLE I. *Ratio of values of Wien to Planck formula spectral energies for a range of* $\lambda\theta$.

$\lambda\theta$ MICRON DEGREES	$(E_\lambda)_w/(E_\lambda)_p$	$\lambda\theta$ MICRON DEGREES	$(E_\lambda)_w/(E_\lambda)_p$
2000	0.9992	7000	0.8712
3000	0.9906	8000	0.8339
4000	0.9721	9000	0.7974
5000	0.9429	10000	0.7619
6000	0.9082	11000	0.7287
		12000	0.6976

[1] I. G. Priest, "A Proposed Scale for Use in Specifying the Chromaticity of Incandescent Illuminants and Various Phases of Daylight," J. O. S. A. **23**, 41 (1933).

[2] D. B. Judd, "The 1931 I. C. I. Standard Observer and Coordinate System for Colorimetry," J. O. S. A. **23**, 359 (1933).

[3] D. B. Judd, "Estimation of Chromaticity Differences and Nearest Color Temperature on the Standard 1931 I. C. I. Colorimetric Coordinate System," J. O. S. A. **26**, 421 (1936).

[4] H. P. Gage, "Color Filters for Altering Color Temperature. Pyrometer Absorption and Daylite Glasses," J. O. S. A. **23**, 46 (1933).

[5] Frehafer and Snow, "Tables and Graphs for Facilitating the Computation of Spectral Energy Distribution by Planck's Formula," Nat. Bur. Stand. Misc. Pub. No. 56, 1925.

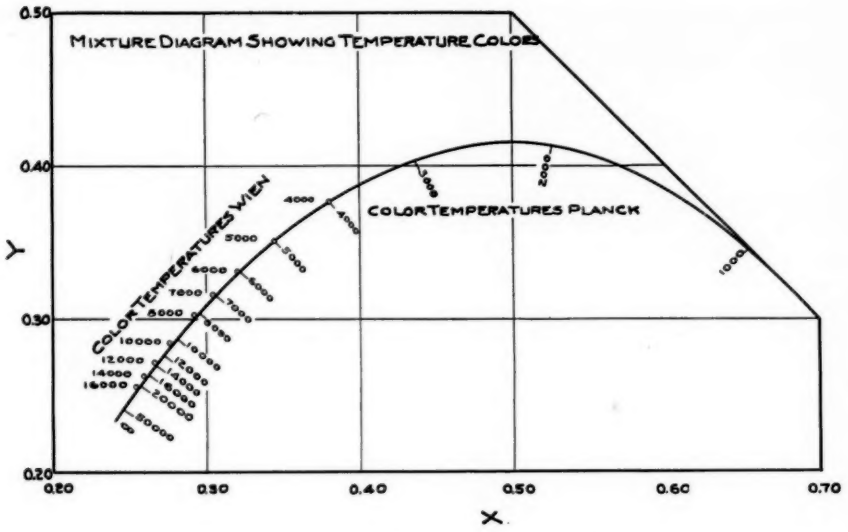

FIG. 1. Mixture diagram showing Wien and Planck temperature colors; x and y are two of the unified tristimulus coefficients.

A table of relative energies $(E_\lambda)_w$ was computed and used to obtain spectral distributions (Wien) for a range of temperatures. The trilinear color coordinates of these distributions were then computed and the color points plotted. These are shown on the mixture diagram, Fig. 1. Where the Wien distribution color points depart materially from the Planckian locus, interpolation between iso-temperature lines was used to obtain the corresponding Planckian color temperatures.

It was found that the coordinates of the color points corresponding to Planckian distributions derived from the Frehafer and Snow data differed slightly from those reported by Judd[3] so that it was necessary to make sure that differences in color temperature referred to measurements made on the same basis. These small variations (approximately one mired) which are probably due to changes in the radiation constants and the thermodynamic temperature scale made since 1925, have no effect on differences at high temperatures, but become important at 4000 and 5000°K, where the difference between the Wien and Planck color temperature scales is so small.

The result of this work is to provide a series of color points derived from the Wien formula and therefore having a Wien color temperature. These points were plotted on the mixture diagram and Planck color temperatures assigned, thus giving a correlation on the basis of subjective color between Wien and Planck color temperatures. The data are most conveniently presented as color temperature differences plotted against Wien color temperatures. Such a plot is shown in Fig. 2 where the differences are plotted on a logarithmic scale because of the range involved. From a large scale plot of the

TABLE II. *Correlation of Wien and Planck color temperatures.*

Color Temperature, K			Reciprocal Temperature, Mireds		
Wien	Planck	Difference	1/ Wien	1/ Planck	Difference
4,000	4,006	6	250.0	249.6	0.4
5,000	5,028	28	200.0	198.9	1.1
6,000	6,075	75	166.7	164.6	2.1
7,000	7,158	158	142.9	139.7	3.2
8,000	8,293	293	125.0	120.6	4.4
9,000	9,512	512	111.1	105.1	6.0
10,000	10,840	840	100.0	92.2	7.8
11,000	12,310	1,310	90.9	81.2	9.7
12,000	13,950	1,950	83.3	71.7	11.6
13,000	15,770	2,770	76.9	63.4	13.5
14,000	17,845	3,845	71.4	56.0	15.4
15,000	20,240	5,240	66.7	49.4	17.3
16,000	22,880	6,880	62.5	43.7	18.8
17,000	25,770	8,770	58.8	38.8	20.0
18,000	28,740	10,740	55.6	34.8	20.8

data a smooth curve was drawn to give the values listed below in Table II.

It is interesting to compare these data with those computed by Gage[4, 6] using as equivalent, Wien and Planck spectral energy distributions which had the same ratio of energy at two well separated wave-lengths, 450 and 650 mμ. Gage's data are indicated in Fig. 2, and show excellent agreement with the results obtained by this more rigorous method which was unavailable when the earlier data were obtained.

The estimation of the accuracy of results of this sort is exceedingly difficult. The deviation of computed points from the curve is small. The agreement with previous data is close. Probably the accuracy of this work is roughly similar to other colorimetric computations made at 10 mμ intervals and carrying 4 significant figures. In Table II the differences can probably be relied upon within ±5 percent.

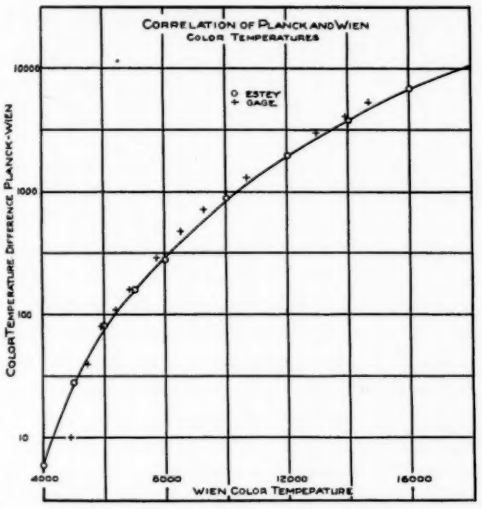

FIG. 2. Correlation of Planck and Wien color temperatures.

It is a pleasure to express appreciation to Electrical Testing Laboratories and to the Spencer Lens Company for their kind cooperation in the preparation of this paper.

[6] H. P. Gage and Norman Macbeth, "Filters for Artificial Daylighting, Their Grading and Use," Trans. I. E. S. **31**, 995 (1936).

Light Signals Sent Around a Closed Path

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IN the Sagnac experiment[1] two simultaneously emitted light signals are sent in opposite directions around a closed optical path, the whole optical system being in rotation about an axis perpendicular to the plane of the apparatus. The two signals upon returning to the point of origin on the apparatus are found to have taken different times, corresponding to velocities of light of $c+r\omega$ and $c-r\omega$, where c is the velocity of light as ordinarily measured by methods involving no rotation, r the radius of the circle described by the observation point and ω is the angular velocity of the light source and observation point as measured by a clock and scale on the supporting platform. The experiment was interpreted by its author as positive evidence for the existence of the luminiferous ether. It has been repeated recently by Dufour and Prunier[2] with the light source and observer separated from and not moving with the apparatus, with an identical result. In a discussion of this experiment Langevin[3] has asserted that by using "local time" on the apparatus the velocity of light is found to be "c." It has previously been dismissed by proponents of the theory of relativity as involving motion of rotation, and as such, along with the gyroscope, capable of explanation only by reference to the influence of all the matter in the universe, i.e., by attaching the pattern of radiant energy to a framework which is not called the ether.

It is the purpose of this paper first to show that the Sagnac experiment in its essentials involves no consideration of rotation, and second to investigate the results obtained when transported clocks are used. It will be shown that while it is true, if we use transported clocks, that the velocity of light measures "c," we must to obtain this result use a separate transported clock for each light beam, with the consequence that at each point of the apparatus we have two coincident clocks, each indicating a different time at the same instant.

For this study we shall consider a modified form of the experiment, the results of which are obviously to be expected to be identical with those obtained by Sagnac. We imagine (Fig. 1) a set of mirrors M_1, M_2, etc. in polygonal arrangement about an inscribed circle with center at 0. A light source S is arranged to move along a chord between two of the mirrors, and a light sensitive device associated with the light source records the emission of the light flash, and also its arrival after reflection around the series of mirrors. For the light signal proceeding in the clockwise direction (designated by the subscript 1) the position of the receiving device when it meets the returned signal is at S_1, for the counter clockwise signal its position is S_2. If t_1 and t_2 are the times of reception of the two signals in terms of absolute space and time (obtained by using measuring rods and clocks which are unaffected by the operations involved in their use) we have the relations:

$$\begin{aligned} P+vt_1&=ct_1,\\ P-vt_2&=ct_2, \end{aligned} \tag{1}$$

where P is the length of the path consisting of the series of chords. From these we have

$$\begin{aligned} t_1&=P/c-v,\\ t_2&=P/c+v. \end{aligned} \tag{2}$$

The difference in arrival time of the two light signals is

$$t_1-t_2=\frac{2Pv}{c^2(1-v^2/c^2)}. \tag{3}$$

Now we can increase the number of mirrors as much as we please, so that the length of the light path will approximate to any desired degree to the circumference of the inscribed circle, although, because of the rectilinear propagation of light, the light path is always a polygon. By moving the light source further from 0 along the radius the chords along which the light travels

[1] Comptes rendus **157**, 708 and 1410 (1913).
[2] Dufour and Prunier, Comptes rendus **204**, 1925 (1937).
[3] Langevin, Comptes rendus **205**, 304 (1937).

approximate to arcs of the circle. We can therefore obviously substitute circular coordinates for the rectangular coordinates thus far used. Doing this relations (2) and (3) become

$$t_1=\frac{2\pi r}{c-r\omega},\qquad t_2=\frac{2\pi r}{c+r\omega} \tag{4}$$

and

$$t_1-t_2=\frac{4\pi r^2\omega}{c^2-r^2\omega^2}=\frac{4\pi(r^2\omega/c^2)}{1-(r^2\omega^2/c^2)}. \tag{5}$$

This last formula agrees with the result of the Sagnac experiment (in which the accuracy attained is only sufficient to show the first order in $r\omega/c$), where the whole apparatus is rotated, and shows, what is indeed obvious by a detailed study of the light paths, that the changes of direction of the several mirrors in the apparatus, produced by the rotation, have a negligible effect on the *length* of the light path. The essential feature is that the light signals are sent around a closed circuit, with motion meanwhile of the receiving device. As a limiting case, let the mirrors coalesce into a circular shell. It is clear that it is then indifferent whether the shell rotates about the center 0, or is stationary. This leaves us with the essential condition that the receiving element shall move with respect to the pattern of radiant energy with which the light signal, which is independent of the velocity of the source, and of the (rotational) velocity of the mirror system, becomes a part.

We now turn to the measurements of arrival time of the light signals which would be made by clocks which suffer a change of rate according to the relation

$$\nu=\nu_0(1-v^2/c^2)^{\frac{1}{2}}.$$

We first consider the modifications of formulae (2), (3) and (4) which occur if the angular velocity of the system is measured by a clock attached to and moving with the light source. Since the time measures less, by the factor $(1-v^2/c^2)^{\frac{1}{2}}$, we have the modified formulae

$$t_1'=\frac{P(1-v^2/c^2)^{\frac{1}{2}}}{c-v},\qquad t_2'=\frac{P(1-v^2/c^2)^{\frac{1}{2}}}{c+v} \tag{2a}$$

and

$$t_1'-t_2'=\frac{2Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}}. \tag{3a}$$

We now proceed to study the values to be obtained if the time of arrival of the light signals is measured by clocks which are transported to the point of reception of the signals, that is by "local time." In order to use the results of previous papers[4] we shall transport these clocks in a carefully specified way, deviations from which in the actual Sagnac experiment can be evaluated as to their significance later. This special procedure consists in carrying the light source and receiving device on a moving band, which travels along the chords between the mirrors[5] with the velocity v. Divisions on this band are contracted by motion in the ratio $(1-v^2/c^2)^{\frac{1}{2}}:1$. The clocks to be used to measure

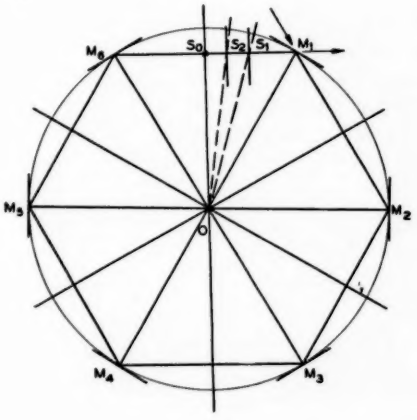

FIG. 1.

[4] J. Opt. Soc. Am. **27**, 263 (1937).

[5] In order to avoid any question of what happens to the clock rates or scale readings on the moving band when it changes direction at each mirror, we can substitute for the single moving band a series of bands moving without change of direction whose paths cross at M_1, M_2, etc. (as suggested by the arrow at M_1 in the figure), at which points the clock and scale readings are transferred from one band to the other.

the arrival times of the light signals at the mirrors and on their return to the source, are transported along this moving band, at some small velocity W, in the direction of travel of the two light signals, that is we have two clocks, of velocities of transport $+$ and $-W$.

We then have, following the procedure of a previous paper,[4] that clocks so transported, compared with a clock at the origin, are set back by

$$\Delta S=\frac{P}{W}[(1-v^2/c^2)^{\frac{1}{2}}-(1-(v\pm W)^2/c^2)^{\frac{1}{2}}]. \quad (6)$$

Solving this by the methods previously used[4] we get for the altered settings of the clocks

$$\Delta S_1=-\frac{Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}}$$

for a clock transported in the clockwise direction,

$$\Delta S_2=+\frac{Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}} \quad (7)$$

for a clock transported in the counterclockwise direction.

The time read by such a clock will be the true time, reduced by the factor $(1-v^2/c^2)^{\frac{1}{2}}$, plus the change in setting. For the time so read for the signal traveling clockwise, by the clock transported clockwise, we have

$$t_R=\frac{P}{c-v}(1-v^2/c^2)^{\frac{1}{2}}-\frac{Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}}=\frac{P}{c(1-v^2/c^2)^{\frac{1}{2}}} \quad (8)$$

and for the signal traveling counterclockwise, by the clock transported counterclockwise

$$t_R=\frac{P}{c+v}(1-v^2/c^2)^{\frac{1}{2}}+\frac{Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}}=\frac{P}{c(1-v^2/c^2)^{\frac{1}{2}}}. \quad (9)$$

We also have that the path P, in terms of the contracted divisions on the moving band measures as

$$P_R=\frac{P}{(1-v^2/c^2)^{\frac{1}{2}}}. \quad (10)$$

We thus have finally, for the measured velocity of either signal

$$\frac{P_R}{t_R}=\frac{\dfrac{P}{(1-v^2/c^2)^{\frac{1}{2}}}}{\dfrac{P}{c(1-v^2/c^2)^{\frac{1}{2}}}}=c \quad (11)$$

which is the result favored by Langevin.

Now in order to find where we stand, having obtained this result, by the use of "local time," let us look at the two clocks which have been brought back into coincidence at the light source, each having traveled around the optical path in the direction of one of the light signals. When we do this we find *each indicating a different time.* Their difference of indication is

$$\Delta S_2-\Delta S_1=\frac{2Pv}{c^2(1-v^2/c^2)^{\frac{1}{2}}}. \quad (12)$$

Looking back at the expression for the difference of arrival times of the light signals, as recorded by a clock traveling with the light source, namely (3a), we find it is *identical* with that just obtained. All we have done by using "local time" is to substitute for the observation of two different arrival times on one clock, the observation of identical arrival times on two clocks, which differ in their settings by just the difference in arrival times shown by the single clock. The performer of the experiment is thus left with the alternative of accepting the observed arrival of the signals at different times as a fact, or, if he must cling to a constant value for the measured velocity of light, of putting his faith in carefully labeled clocks which tell him the signals arrive at the same time. But he must avoid looking at both[6] clocks at once! In short

[6] There are of course not merely two clocks, but an infinity of clocks, when we include those which could be transported at finite speeds, and around other paths. As emphasized previously the idea of "local time" is untenable, what we have are *clock readings*. Any number of clock readings at the same place are physically possible, depending on the behavior and history of the clocks used. More than one "time" at one place is a physical absurdity.

the physical fact cannot be evaded by juggling the measuring instruments.

In conclusion the relationship of the experiment here discussed, with its stationary polygonal arrangement of mirrors, and its moving band, on which the measurements are made, to the actual Sagnac experiment may be briefly considered. Let us reduce the Sagnac experiment to the rotating cylindrical mirror shell above suggested, and let the light source be carried on a chord of that shell. In order to go over to this case we need to know what contraction of dimensions, if any, the shell experiences on rotation, and what change of rate a clock experiences when transported over a curved path. Without knowing these we can nevertheless from general considerations predict that the phenomena will be altered from those we have studied only by second-order times in v/c or $r\omega/c$. If our path length is measured by divisions on the ring these divisions will be contracted exactly as the ring, so that the measured path length will always be P, instead of $P/(1-r^2\omega^2/c^2)^{\frac{1}{2}}$ as obtained from the moving band. With regard to the true times of travel of the light signals, these will be directly affected in proportion to the contraction of the ring, which from consideration familiar in connection with the Ehrenfest paradox must be less than $(1-r^2\omega^2/c^2)^{\frac{1}{2}}$. The alteration of clock rate by transport in a curved path can only differ from $(1-r^2\omega^2/c^2)^{\frac{1}{2}}$ by small terms involving $1/r$ since as r is increased it must approximate to $(1-r^2\omega^2/c^2)^{\frac{1}{2}}$ as a limit. We can therefore always insure, by making our apparatus large enough, that the latter factor is substantially correct. It is therefore apparent that our results must hold at least as far as the first order of $r\omega/c$ for the actual Sagnac experiment.

The net result of this study appears to be to leave the argument of Sagnac as to the significance of his experiment as strong as it ever was. The suggested use of "local time" merely offers another way of measuring the effect of rotating the apparatus, namely in terms of the differences between two clocks carried around a circuit, instead of differences of arrival time of two light signals sent around the same circuit. The rotation, which can be measured in either of these ways, is not relative rotation of the apparatus with respect to the platform on which it is mounted, or to the laboratory—either of these might be rotated with respect to the apparatus, with no resultant Sagnac effect. The observer on the apparatus has just one reference framework by which he can predict whether the Sagnac effect will appear or not; that framework is the pattern of radiant energy from the stars. If his apparatus rotates with respect to the stars he will observe a Sagnac effect, if it does not, then no matter how great relative rotation it exhibits with respect to its material surroundings, there will be no Sagnac effect.

The Experimental Determination of the Optical Constants of Metals. The Methods and Results*

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INTRODUCTION

THE optical properties of bulk metals are generally characterized by two constants, namely, the refractive index n, and the index of absorption, k. A third constant, namely, the reflectivity for normal incidence, R, may be determined directly by experiment, or computed from a knowledge of n and k. While there has been little ambiguity associated with the meaning of the refractive index and the reflectivity, there has been considerable confusion associated with the index of absorption. The latter gives the relation between the amplitudes of the light wave in the metal, one wave-length apart, the relation being $E_2=E_1e^{-2\pi nkx/\lambda_0}=E_1e^{-2\pi kx/\lambda_m}$. In this equation λ_0 is the wave-length of the light in the vacuum (the beam traveling in the x direction), and λ_m is the wave-length in the metal. Hence $(1/2\pi)\times\log(E_1/E_2)=nk$ or k, depending upon whether x is to be taken equal to the wave-length in vacuum or in the metal, respectively. The literature is very much confused on this point, and there is even no agreement upon the name to be applied to nk or k. The product nk is often referred to as the extinction coefficient. Only where authors have clearly stated the formulae that have been used, has this unnecessary confusion been avoided.

Generally speaking, the field of metal optics occupies a comparatively quiet sector of physics. Here there is no feverish concentration of research activity. There are no million-volt machines to contend with as in the study of nuclear phenomena, nor do we have the grand pyrotechnical displays of electrical discharges in gases. If the study of the optical constants happens to be in the visible part of the spectrum, likely as not the observer is sitting quietly with his eye "glued" to an eyepiece, calmly twirling an analyzing Nicol.

None the less, a knowledge of the optical properties of metals is of importance not only as shedding light on the intrinsic properties of the metals, but also as affording raw material for the theorist to work with. In the early years of this century, the favorite occupation initiated by Drude and Schuster was the computation from the optical constants of the number of "free" electrons per atom of the metal. More recently in the hands of Ives, Fry, Fleischer, and Dember, the optical properties of the metals have been utilized to account for their photoelectric behavior. The determination of optical constants has also received a quickening impulse as the result of the application of quantum mechanics to the metallic state by Kronig, Mott, A. H. Wilson and others. On the other hand, since the optical behavior of a metallic surface may be markedly influenced by surface films, a study of the optical properties of metals suspected or known to possess surface films becomes of great importance to the metallurgist.

In outlining the general experimental methods and results that have been obtained, I shall confine myself mainly to the ultraviolet and visible portions of the spectrum, in order to keep this paper within the time allowed. If one examines the broad literature covering the field of metal optics for the past fifty years, one becomes impressed by the scope and variety of the methods of measurement of the optical constants. I have kept a bibliography more or less complete, of the results published on optical constants of metals and associated subjects, and I find on my list no fewer than four hundred references. Broadly speaking, the methods can be divided into two general classes, namely, those that depend upon direct reflection and transmission measurements, and those that depend upon the analysis of the state of polarization of the light reflected from the surface of the specimen. The refraction methods which are so applicable to transparent insulators are un-

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available for metals, or applicable only to a very limited extent, due to the very high absorption of light in metals. Transmission measurements are of course impossible with opaque layers of metals, but are of great importance in the study of optical properties of transparent or semi-transparent films. Very thin metallic films possess optical and electrical properties different from those of the metal in the bulk, but here again, due to the time limitations, I shall confine myself to layers of metals which are opaque to the radiation.

The Polarimetric Method

The polarimetric method for measuring the optical constants consists essentially in allowing monochromatic light polarized in a given plane with respect to the plane of incidence, to fall at a given angle upon the metal surface, and then determining the nature of the elliptically polarized light that is reflected. n and k are evaluated from the constants of the ellipse by formulae which are to be found in advanced treatises on light or in the literature.[1] The constants of the ellipse are expressible in terms of the phase difference Δ of the components of the light polarized in and perpendicular to the plane of incidence, and the azimuthal angle between the restored plane of polarization and the plane of incidence. The dependence of the phase and azimuthal angles upon the angle of incidence and nature of the reflecting surface, is illustrated by Fig. 1 for iron, hematite and magnetite.[2] The last named substance yields graphs which are essentially those obtained for an insulator, while the graphs for iron are the characteristic ones of a metal.

The phase angle may be measured by a compensator of the simple Babinet or the Soleil-Babinet type, or by a quarter-wave plate. The azimuthal angle is determined from the position of an analyzing device such as a Nicol prism. There have been many ingenious devices described by various workers for the accurate determination of the extinction point. This involves some half-shade device in the eyepiece,

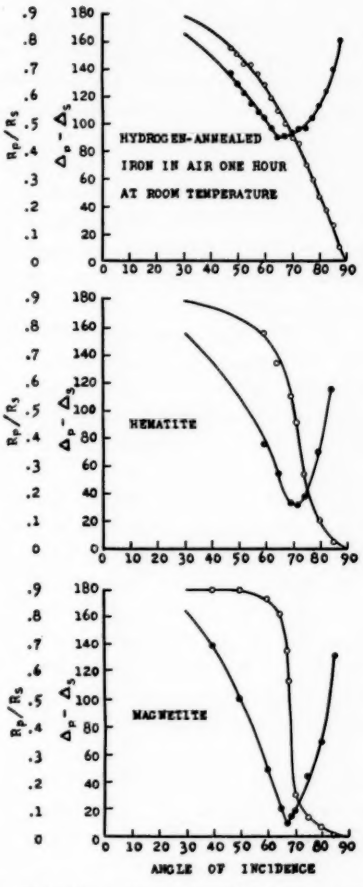

Fig. 1. Variation with angle of incidence of the phase difference (shown by clear circles), and tangent of azimuthal angle (black circles) of restored plane of polarization of iron, hematite and magnetite.

such as, for example, the smoked glass described by Zehnder,[3] or a pair of glass plates of variable inclination such as described by Kent,[4] or the doubly refracting plates of Brace.[5]

In the ultraviolet region, either a fluorescent screen in the eyepiece might be utilized, or what has been found more effective and accurate is to obtain a photographic record of what one would see if working in the visible. This means that a succession of photographs would have to be taken for various positions of the compensator and analyzer, and the extinction position evalu-

[1] J. B. Nathanson, J. O. S. A. **27**, 393 (1937).
[2] The author is indebted to C. E. Leberknight and B. Lustman of the Carnegie Institute of Technology for these graphs.
[3] L. Zehnder, Ann. d. Physik **26**, 985 (1908).
[4] C. V. Kent, Phys. Rev. **14**, 459 (1919).
[5] D. B. Brace, Phys. Rev. **18**, 70 (1904).

ated from the plates by means of some form of microphotometer. This would represent quite a laborious process, but Voigt[6] has shown how all of the data necessary might be obtained from one photographic plate.

Voigt's method has been recently applied by Ives[7] and Briggs in the measurement of the optical constants of the alkali metals. In this method (Fig. 2) a pair of prisms, one of right and the other of left rotary quartz, combined to form a composite plate with the crystalline axis of each perpendicular to the face of the plate, is placed between the Babinet compensator and the analyzer. The light passing through certain portions of the compensator is rendered plane polarized, and upon passing through the composite rotary plate has its plane rotated through different angles depending upon the relative thicknesses of the right- and left-handed quartz traversed. As a result, the customary system of parallel fringes is replaced by a composite spot pattern, which is photographed on a piate. The phase difference and azimuthal angle are evaluated from the horizontal and vertical shift of this spot pattern relative to that obtained without the metallic specimen.

One other method employed in the ultraviolet and visible regions might be mentioned, namely, the interference and reflection method of Malsch.[8] The method of Malsch is based upon two relations connecting the optical constants with the experimentally determined reflectivity and the absolute phase change of the light incident normally on the mirror. The reflectivity is given by $R=((n-1)^2+n^2k^2)/((n+1)^2+n^2k^2)$ while the phase change Δ is given by $\Delta=-2nk/(n^2(1+k^2)-1)$. The phase change on reflection Δ, for light incident normally, was determined from observation of the interference fringes obtained from a small angle air prism formed between the mirror surface and a thin quartz plate. Though Malsch claims that his method requires no polarization apparatus in the ultraviolet, nevertheless the phase difference obtained has to be evaluated in terms of that obtained in the visible region by employing a polarization

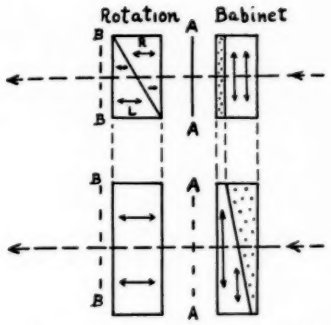

FIG. 2. Voigt's rotary quartz prism combination and Babinet compensator. Side and plan views shown by upper and lower figures, respectively. *AA* represents fringe system observable with compensator alone; *BB* represents spot system observable for light transmitted through entire prism train.

method for measuring the principal angles of incidence and of azimuth.

PHOTOELECTRIC DETERMINATIONS OF THE CONSTANTS OF ELLIPTICALLY POLARIZED LIGHT

It is to be expected that the field of metal optics would not be immune to the invasion of the photoelectric cell. In the past five years some attempts have been made to substitute the photo-cell for the eye. This is not only desirable in order to relieve eyestrain, but also particularly useful in the ultraviolet and near infra-red portions of the spectrum. Bruhat and Guinier[9] seem to be the pioneers in this field. Quite recently, Bor[10] employed a photo-cell connected to a Lindemann electrometer as a substitute for the customary eyepiece, to determine the correct positions of the Soleil-Babinet compensator and Nicol analyzer for maximum extinction of the light. However, a more constant source of light is essential than that obtained by use of the visual method. Bor claims that the results compare favorably with those obtained visually.

An interesting variation is that due to Kent and Lawson,[11] whose scheme consists in converting the elliptically polarized into circularly polarized light by means of a fixed phase plate of mica or selenite. The circularly polarized light is passed through a Nicol prism revolved at

[6] W. Voigt, Physik. Zeits. **2**, 303 (1901).
[7] H. E. Ives and H. B. Briggs, J. O. S. A. **26**, 238 (1936); **27**, 181 (1937); **27**, 395 (1937).
[8] J. Malsch, Ann. d. Physik **7**, 360 (1930).
[9] G. Bruhat and A. Guinier, Rev. d'optique **12**, 396 (1933).
[10] J. Bor, Nature **139**, 716 (1937).
[11] C. V. Kent and J. Lawson, J. O. S. A. **27**, 117 (1937).

40 r.p.s., and then is incident on a photo-cell. The adjustment of the compensator plate is then said to be the correct one for which no modulation of the amplified output of the photo-cell can be detected. The advantages of this method are that stray unpolarized light has no effect on the setting, and that one does not have to work with very feeble light intensities near the extinction point.

Preparation of the Surface

The first step in the determination of the optical constants of an opaque specimen of a metal is the preparation of the surface. The aim of course is to obtain a very highly polished surface, as free of scratches as possible, and which surface shall be characteristic of the metal itself, and not of the mode of polish. About this point there has been much discussion and disagreement, from the early work of Drude to this date. Undoubtedly much of the disagreement of published results can be attributed to just this factor. We thus face experimental difficulties that are always inherent in connection with the study of surface phenomena, such, e.g., as work functions. Whereas fresh surfaces of transparent crystals can be obtained by splitting of the crystal, and clean liquid surfaces can be obtained by a continual flow method, no such recourse can be had in the case of solid metals. In x-ray work, the comparatively deeper penetration of the radiation in a metal means that a fairly thick layer of the material enters into the control of such optical properties as the reflecting power. However, in the so-called optical range, the complete absorption of the light occurs over a distance of a few wave-lengths or less, so that the influence of a surface layer in this spectral range is of far greater importance than in the x-ray spectrum.

Since light can penetrate a metal to only a very small thickness, it would seem that a lighter but effective polish should be more favorable for obtaining a true optical surface than a more rigorous polish whose effects might penetrate deeper into the metal, thus producing a sort of strained layer. It must be remembered that metallic reflection will depend not only upon the intrinsic properties of the atoms, but also on their arrangement after polishing. The thin film produced by polishing is of some 50 angstroms in thickness, and was originally investigated by Sir George Beilby, who claimed that the layer is formed by the flowing of the metal during the act of polishing to fill the minute crevices and valleys of the surface. About the existence of this layer there seems at present no doubt, but there is some disagreement as to its structure. Beilby has claimed that this layer is amorphous, which claim has been upheld by the electron diffraction observations of G. P. Thomson,[12] who detected no regular diffraction pattern. This was, however, challenged by Randall and Rooksby,[13] and French,[14] who claimed that some crystalline structure remained, but that the act of polishing had reduced the sizes of the crystals to such low values that the diffraction rings had become diffuse. The real situation is probably somewhere between these two extremes, the Beilby layer consisting of a mixture of amorphous layers in which might be imbedded very minute or submicroscopic crystals.

The most noteworthy attempt to obtain a highly polished surface freed of the Beilby layer is that by Lowery, Wilkinson and Smare.[15] Using a sample of electrolytic copper, they first polished the surface in the usual manner, i.e., by a succession of finer emery papers, followed by polishing on a wheel with a paste of alumina, until all scratches were removed, and presumably a Beilby layer formed. The strains introduced in the surface by the cold working were then relieved by annealing in a furnace under a hydrogen atmosphere, after which the Beilby layer was gradually removed by an electrolytic method devised by Jacquet.[16] In this electrolytic method of polishing, the copper sample is made the anode of a cell in which the electrolyte is a solution of orthophosphoric acid. By this process as much of the Beilby layer can be removed as desired, depending upon the length of time devoted to the electrolysis.

Alkali metal surfaces must necessarily be pre-

[12] G. P. Thomson, Proc. Roy. Soc. **A128**, 649 (1930).
[13] J. T. Randall and H. P. Rooksby, Nature **129**, 280 (1932).
[14] R. C. French, Nature **129**, 169 (1932).
[15] H. Lowery, H. Wilkinson and D. L. Smare, Phil. Mag. **22**, 769 (1936).
[16] P. Jacquet, Nature **135**, 1076 (1935).

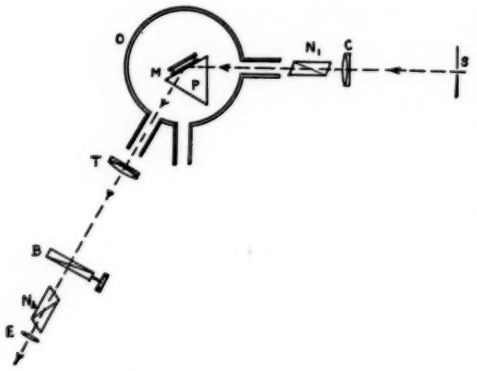

FIG. 3. Arrangement for measuring optical constants of an alkali metal enclosed in a glass cell M, placed in optical contact with prism P. Temperature variation may be produced by electrically heated oven O.

served under vacuum or in an inert atmosphere. If one attempts to study these metals in the liquid state, as Drude did for sodium in 1898, the troublesome question of surface is not so serious, for the liquid metal naturally takes up a very smooth surface. And if one employs high vacuum technique, the question of foreign films may not be so serious. The real difficulty enters when one wishes to study these metals in the solid state. Alkali metals upon solidifying exhibit surfaces which are not very smooth, due to crystallization effects, although Ives and Briggs[7] obtained good optical surfaces of sodium and caesium by careful annealing and cooling of the metals. In despair, the Duncans[17] in 1913 followed by Nathanson,[18] and Ives and Briggs,[7] have made observations on alkali metal surfaces obtained by condensing or pressing the liquid metal while under vacuum, on a glass or fused quartz surface. With the aid of suitable glass prisms in optical contact with the glass plate of the cell M, Fig. 3, observations are thus made on the glass-metal interface. The angle of incidence may be varied by employing prisms, P, of different angles. Whereas the index of absorption thus obtained has been assumed to be independent of the nature of the transparent medium in contact with the metal, the observed refractive index of the metal, when multiplied by that of the transparent medium, yields the absolute refractive index of the alkali metal.

Instead of polishing the bulk metal, opaque layers of metals have been obtained directly by deposition on a glass or quartz plate by cathodic sputtering or by condensation from the vapor phase. The last named method as utilized by O'Bryan[19] and Lowery and co-workers[15] seems the most promising, in view of the recent improvements in the technique of the vaporization of metals. With suitable preliminary outgassing and working under good vacuum conditions, the opportunity for obtaining uncontaminated metallic surfaces should be excellent, particularly since O'Bryan also made his measurements on the metals while in place under vacuum conditions.

The method of producing a metal surface by cathode sputtering is a very convenient one, and has been employed to obtain semi-transparent films of the metals. The rate of sputtering is sufficiently high for only a few of the metals to enable one to obtain an opaque layer within any reasonable time. The sputtering process necessarily requires the presence of a gas, thus running the danger of a possible chemical reaction between the gas and the metal. Even though an inert gas is generally employed, under the bombardment of positive ions, complex physical conditions may arise to favor unusual chemical reactions resulting in the formation of hydrides and nitrides. This is illustrated by Fig. 4 of a photograph of a semi-transparent film of variable thickness of platinum deposited upon a Pyrex glass plate by sputtering in hydrogen. The platinum cathode consisted of a disk only 1.5 cm in diameter. When viewed by reflected monochromatic light, the film exhibits a regular interference pattern of circular fringes,[20] which pattern is markedly altered in appearance when the film is heated to about 250°C, as shown by the left side of the photograph.

VARIATION OF OPTICAL CONSTANTS WITH NATURE OF PREPARATION OF SURFACE

There is apparently a difference between a real heaviness of polishing such as burnishing, and

[17] R. W. and R. C. Duncan, Phys. Rev. **36**, 491 (1913).
[18] J. B. Nathanson, Phys. Rev. **25**, 75 (1925).
[19] H. M. O'Bryan, J. O. S. A. **26**, 122 (1936).
[20] J. B. Nathanson, J. O. S. A. **23**, 388 (1933).

a more gentle but good polishing of the surface, which does not permanently alter the optical characteristics of the surface. Lowery and Moore[21] found in studying the relation between the state of strain of a copper surface and its optical constants, that an increased heaviness of polish was accompanied by a decrease in the absorption index and reflectivity, and by an increase in the refractive index. These results are not entirely in agreement with the older work of Tool,[22] who stated that polishing will tend to increase all of the constants, though n will be affected much less so. By the same token, incomplete or matt polish will tend to decrease the values of these constants, particularly k and R.

Margenau[23] studied the effect of surface strains on the position of the sharp dip in the reflectivity curve of silver at 3160 angstroms. He found that by relieving these strains by a light etching or heat treatment, the dip in the curve shifted to smaller wave-lengths by about 20 angstroms. This could be accounted for by assuming that polishing produced a slight increase in the refractive index in the visible region of the spectrum. Similar results may be found upon examination of Drude's[24] older work on silver and copper. The surfaces of these metals when first highly polished with steel wool and then gently glowed, exhibited a slight decrease in n and an increase in k and R. As would be expected the electrolytic method of Lowery, Wilkinson and Smare[15] for polishing copper surfaces yielded higher values of k and R and lower values of n than those obtained by mechanical polishing.

It would therefore seem from an examination of the results of various workers that given a series of surfaces of a given metal polished in different ways, that surface is optically most characteristic of the metal for which the extinction coefficient and the reflectivity are the largest, and the refractive index either the smallest or else altered only slightly.

It is interesting, however, to note that contrary to our expectations, the optical constants of surfaces prepared by vaporization in a vacuum were found by Lowery and co-workers to be somewhat intermediate between those prepared mechanically and those prepared by their special electrolytic method of polishing. Comparing the results of O'Bryan on vaporized surfaces of magnesium, aluminum and manganese with those of other investigators on these same metals obtained by polishing of the bulk metals, only aluminum seems to meet the previously mentioned requirements for a characteristic optical surface. This might be due to a granular structure of the evaporated surface which is similar to the inclusion of minute crystals in a Beilby layer on a mechanically polished surface.

Similar results may be expected with a cathodically sputtered surface, where the inclusion of the residual gas in the metal layer might distort the natural crystalline structure, producing effects similar to those of a highly strained surface. As a matter of fact, Summers[25] found that a mechanically prepared surface of molybdenum yielded a higher value of all of the optical constants than a sputtered surface of the same metal.

Irrespective, however, of the method of preparation of the optical surface, the general trend of the dispersion curves of these optical constants will be about the same. With this conclusion there seems to be agreement among the workers in this field, as illustrated most strikingly by the work of Lowery and co-workers on copper.

FIG. 4. Circular interference fringes of wave-length 5461A, in a semi-transparent film of platinum produced by sputtering in hydrogen. Faded portion on left due to heating left side of film at 250°C.

[21] H. Lowery and R. L. Moore, Phil. Mag. **13**, 938 (1932).
[22] A. Q. Tool, Phys. Rev. **31**, 1 (1910).
[23] H. Margenau, Phys. Rev. **33**, 1035 (1929).
[24] P. Drude, Ann. d. Physik **39**, 514 (1890).
[25] R. D. Summers, J. O. S. A. **24**, 261 (1934).

Variation of Optical Constants with Temperature

Theories of the metallic state indicate that there is an intimate relation between the electrical conductivity and the optical constants. Since the electrical conductivity of a metal is known to be a function of the temperature, we should therefore likewise expect to find that the optical constants would possess a temperature coefficient. The well-known work of Rubens and Hagan seems to substantiate this for the infrared. In the visible part of the spectrum, however, the results are not so definitely established. The original experiments of Kundt[26] with metallic prisms seemed to show that there was a definite increase in the refractive indices of silver, antimony, copper, platinum, iron and nickel for temperature variation from room to about 110°C. Several years later, A. Pflüger repeated Kundt's experiments, but found no variation, and he assigned Kundt's results as due to errors in collimation. Furthermore Drude[27] never considered his results for temperatures up to 200°C as being sufficiently conclusive to prove a temperature variation. This lack of temperature dependence of the optical constants seems to have been substantiated by most of the subsequent workers in this field. For example, Aster[28] in studying the optical constants of lead, tin, bismuth, and Wood's alloy in their molten state for λ6020A, found no variation in their optical constants for temperature variations of several hundred degrees.

If the claim is made that perhaps a definite variation with temperature might be observed by extending the range of temperatures employed, then we encounter some contradictory results. From observations on the emissivity of tungsten for temperatures extending to 2250°K, and the degree of polarization of reflected light of 6650A, Worthing[29] claimed that there was some temperature dependence even though the optical constants did not vary markedly with temperature. On the other hand, Schubert[30] came to the conclusion from observations of platinum extending to 900°C, that a true temperature effect could not be claimed with any true assuredness. It is true that n, k, and R seemed to be larger for the higher temperatures than for the lower ones, but the constants at room temperature seemed to change each time the metal was cooled. Schubert believed that any changes in optical constants which he had observed might be assigned to changes in surface conditions and to recrystallization effects. This is not very surprising in view of the previous discussion on the difficulty of obtaining a true optical surface. Schubert's results are thus in agreement with the much older observations on platinum by P. Zeeman,[31] and v. Laue[32] and Martins, whose investigations extended up to 800°C and 1500°C, respectively.

From all the aforesaid we must conclude that the optical constants for the ultraviolet and visible regions of the spectrum apparently vary very little or not at all with temperature. It may be that these constants, like the specific heats of metals at ordinary temperatures, are insensitive to changes in temperature. Any attempt, however, to make further investigations of this problem will have to entail some method of maintaining a surface whose physical condition will not be altered with change in temperature.

Variation of Optical Constants with State

A study of the literature reveals the fact that only for comparatively few metallic elements have the optical constants been determined for both the solid and liquid states. The metals about which we have such information are bismuth, lead, cadmium, tin, mercury, potassium, caesium, and gallium. Though there are differences in most cases in the optical constants of a metal for the two states, they may not be true ones, for the observations may not have been taken on the same metallic surface. In view of the previous discussion, this is quite pertinent. An analysis of Morgan's[33] results with the identical potassium surface reveals a slight lowering in the values of n and k as the metal changes from a solid to liquid state. Haak and

[26] A. Kundt, Wied. Ann. **36**, 824 (1889).
[27] P. Drude, Wied. Ann. **39**, 538 (1890).
[28] A. K. Aster, Phys. Rev. **20**, 349 (1922).
[29] A. G. Worthing, J. O. S. A. **13**, 635 (1926).
[30] F. Schubert, Ann. d. Physik **29**, 473 (1937).
[31] P. Zeeman, Comm. Lab. Leiden, No. 20 (1895).
[32] M. v. Laue and F. Martins, Physik. Zeits. **8**, 853 (1907).
[33] R. Morgan, Phys. Rev. **20**, 203 (1922).

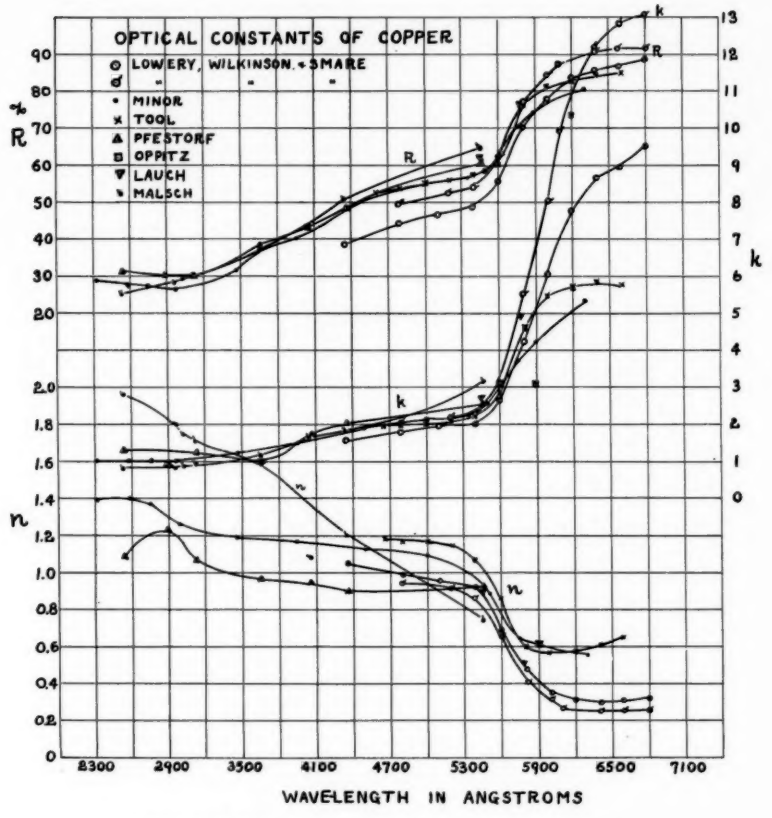

FIG. 5. Collected results on optical constants of copper.

Sissingh[34] working with a mercury mirror found no abrupt change in the phase difference of the two components of the reflected light as the temperature was reduced to −80°C. They concluded that optically speaking, solid and liquid mercury are identical.

In order to be certain of the same optical surface in the two states, it is desirable to have the surface protected from chemically active gases. Metals such as the alkalis, gallium, or mercury, whose solidification temperatures are not too high, are naturally the most convenient surfaces with which to work. I have tried potassium[35] and caesium, and have found no appreciable change in optical properties which could be assigned to a change in state. Lange[36] has determined the optical constants of solid gallium. Working about the same time, I have found the values of n and k for liquid gallium[37] to be considerably different from Lange's results for solid gallium. Interestingly enough, however, the products of n and k seem to be about the same for various wave-lengths in the visible spectrum, as those given by Lange. Drude[38] gave results for tin, which also showed a change in the optical constants with change in state, but I cannot tell from his original account whether or not observations were made on the same tin mirror. There is thus insufficient experimental evidence to warrant any general conclusion

[34] J. J. Haak and K. Sissingh, Proc. K. Akad. Amsterdam **21**, 691 (1919).
[35] J. B. Nathanson, J. O. S. A. **17**, 343 (1928).
[36] H. Lange, Zeits. f. Physik **94**, 650 (1935).
[37] J. B. Nathanson, Phys. Rev. **49**, 887 (1936).
[38] P. Drude, Ann. d. Physik **39**, 481 (1890).

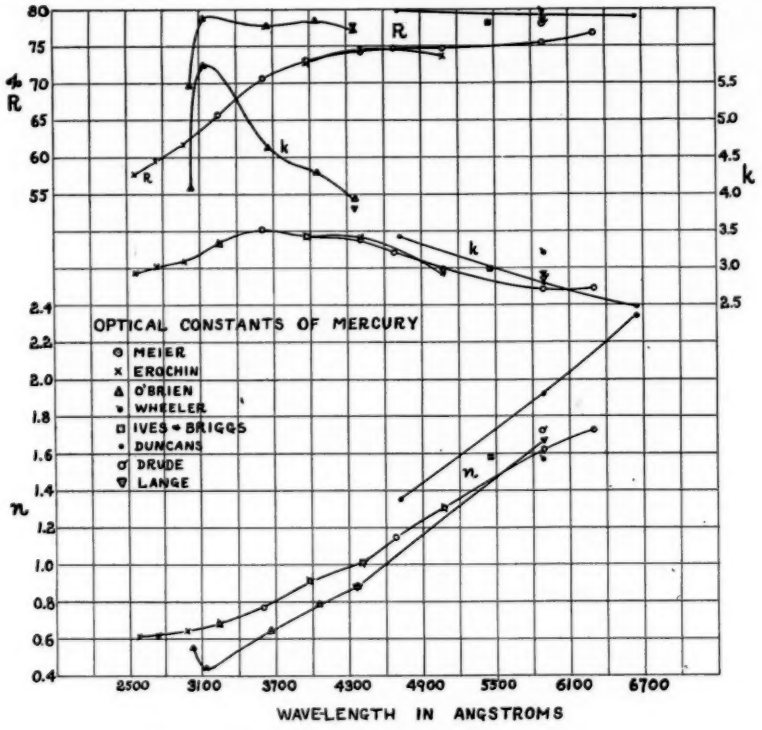

FIG. 6. Collected results on optical constants of mercury.

relative to changes in optical constants of metals with change in state.

VARIATION OF THE OPTICAL CONSTANTS WITH WAVE-LENGTH

In comparing the variation with wave-length of the refractive indices of insulators with those of metals, the striking feature that is to be noted is the relatively small percent change in these indices for insulators as compared to the relatively large percentage change for metals. Whereas the refractive indices of insulators generally decrease with increase of wave-length, barring absorption bands, in the case of metals the more general tendency seems to be in the opposite direction. However, there are some exceptions, the most notable ones being the alkalis, and to a lesser extent the alkali earths. As far as the index of absorption is concerned, the division among the metallic elements is fairly even, about as many possessing an increasing k as a decreasing one with increase in wave-length. So far as the reflectivity is concerned, we can say with greater definiteness that it increases with increase in wave-length, which tendency as is well known, is particularly established in the infra-red.

Another general tendency which might be mentioned, and which has been noted by investigators, is that though the absolute values of the optical constants of a given metal might vary considerably as given by different investigators, the forms of the dispersion curves are quite similar. This is illustrated by Fig. 5 showing the collected results for copper. I have chosen this metal because it has been investigated so much in various laboratories; it is quite susceptible to oxidation so that a large variation in the state of the surface might be expected; and because its dispersion curves are interesting.

It has been assumed that if a surface were more easily reproducible, the agreement between the results of different investigators would be

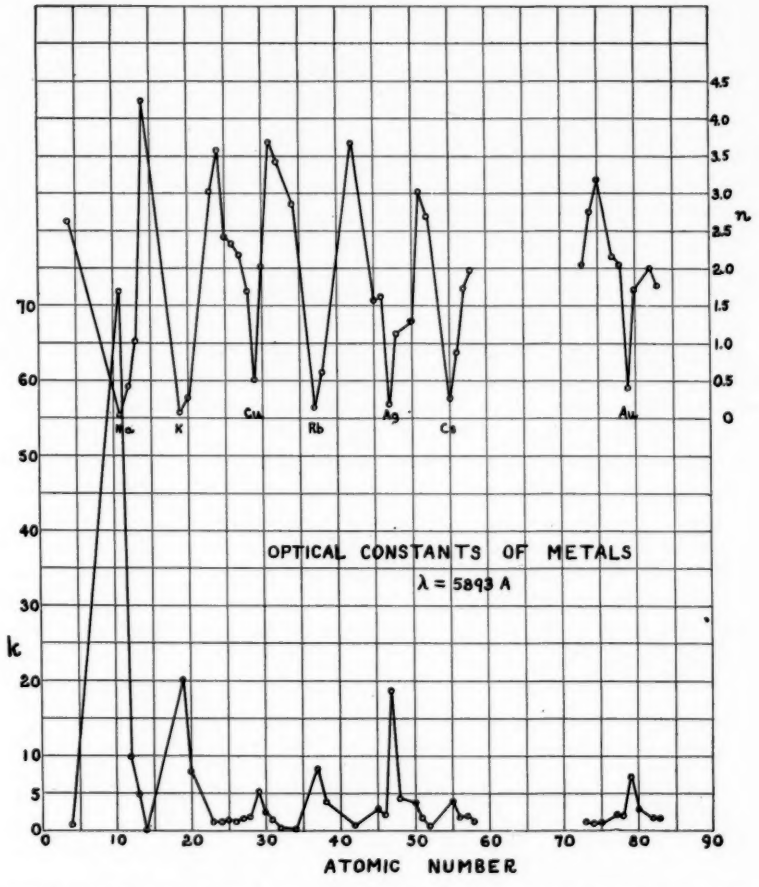

FIG. 7. Variation with atomic number of the optical constants of metals for sodium yellow light.

much better. Such a surface would most reasonably be supplied by mercury, whose dispersion curves are shown in Fig. 6. It must be admitted that the results are disappointing, since the agreement is not particularly superior to that obtained for copper. Again the form of the dispersion curves are similar. It is true that Meier's and Erochin's results for mercury are in excellent agreement, but it must be borne in mind that their work was performed in the same laboratory, the same apparatus being employed. With all due credit to any laboratory, the results obtained by different investigators working on the identical problem but with different methods and apparatus, are always of greater significance. In this connection might be mentioned the collected results of different investigators on the alkali metals as graphically shown by Ives and Briggs.[7] The agreement seems to be best for sodium and rubidium.

VARIATION OF OPTICAL CONSTANTS OF METALLIC ELEMENTS WITH THEIR ATOMIC NUMBERS

I should not like to leave too much of a pessimistic impression with you as regards the values of the optical constants. A brighter picture is obtained if one examines how these constants vary with the atomic number, keeping the wavelength constant. Fortunately for our purpose, a sufficient number of metallic elements have been

studied. From the data given in the *International Critical Tables*, Vol. V and that published since the first issue of these tables, I have plotted the average results of the optical constants of the various metals for yellow sodium light. The plot which is shown in Fig. 7 is most interesting as affording another striking confirmation of the existence of a periodic variation in the physical properties of the elements. It will be noticed with what regularity the refractive indices dip down to a minimum for the metals in group I of the periodic table, while the indices of absorption rise to a peak for the same group. The gap for atomic numbers 59 to 72 inclusive is occupied by the so-called rare earth metals, which have as yet not been studied. The variation between extreme values of these constants is a maximum for low atomic numbers and decreases somewhat for the higher atomic numbers.

While the refractive indices of insulators are always greater than unity, it is quite the common thing to find metallic elements with indices less than unity, particularly those in the first and second groups of the periodic table as shown in Fig. 7. It must be remembered, of course, that a refractive index less than unity means that the wave or phase velocity, and not the signal velocity, is greater than the velocity of light in vacuum. The variation with atomic number of the optical constants of the alkali metals shows the greatest regularity. It is also to be noticed that low refractive indices are generally associated with high absorption indices and *vice versa*. The highest refractive indices are possessed by silicon, vanadium, chromium, gallium (solid), germanium, molybdenum, antimony, and rhenium, these metals being in the middle groups of the periodic table.

Any comprehensive theory of metallic dispersion must take the previously stated facts into account. We do possess a large mass of data on optical constants, some apparently unrelated. However, previously to the Bohr-Sommerfeld theory of spectral lines and atomic structure, spectral data also seemed confused, vast, and unrelated. We now need a Bohr-Sommerfeld theory relating optical constants of metallic elements with their atomic constants.

Determination of Optical Constants with the Aid of an Optical Pyrometer*

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(Received April 28, 1938)

FEW seem to recognize the possibility of using an optical pyrometer in determining optical constants. Yet it was with such apparatus that it was first shown that, in the case of certain metals, there are changes in their optical constants with changes in temperature and that some of the changes have been measured. As presented, the most convincing evidence for the existence of changes with temperature consists of observed changes in the spectral emissivity and the spectral reflectivity in the visible region in the cases of W, Mo, Ta, Au, Pt. That these changes, in the case of tungsten, may have been due to the presence of an inert atmosphere as seems indicated by the work of Wahlin and Whitney,[1] is not of great concern in our discussion of method. Moreover, most others, using the more standard method, have worked under somewhat similar conditions as to the atmosphere in which their specimens were immersed. So far as actual measured changes in optical constants with changes in temperature, the writer[2] knows only of his own measurements for tungsten, wherein he obtained for room temperature 3.86 and 0.81 as values for n and k, and 3.85 and 0.89 as values for 1900°K.

Apparatus convenient for use with the optical pyrometer method is shown in Fig. 1. Specimen data obtained by reflection from tungsten at room temperature, which has been used in determining angles of principal incidence and of principal azimuth, are shown in Fig. 2.

In view of the foregoing, some comment as to special features relating to the method may be appropriate. They are:

(1) Optical pyrometer measurements may be conveniently made on small filaments which may, in turn, be conveniently heated to and maintained at definite high temperatures for purposes of outgassing, freeing the surface of the specimen from impurities, and the carrying out of the tests relating to temperature variation.

(2) For reflection measurements, the precision of polish presumably needs to be as great for the optical pyrometer method as for the more common method. With the former method, however, the determination of satisfactoriness of conditions is relatively very simple.

(3) For optical constant determinations by means of emission measurements, there is needed a generally shiny surface only. Actual polish with its possible contamination is of but little help. From a slightly uneven surface, as one can see (Fig. 3), there is no possibility that the viewer shall be troubled by brightness building in consequence of added reflections or, on the other hand, that he shall see, "as it were," "radiation holes" in the beam entering the optical pyrometer.

(4) The accuracy of setting of the optical pyrometer is of the same order as those of other methods.

(5) When using an optical pyrometer, one actually looks at the spot of the specimen which is being investigated.

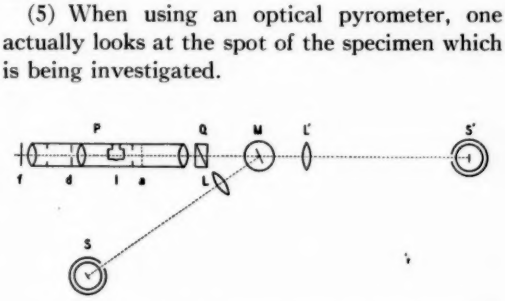

FIG. 1. Diagram showing apparatus used in measuring the two principal polarized light components in the beams emitted or reflected by polished metal ribbon filaments for various angles of emergence or of incidence. S is a nearly enclosed light source, M a lamp containing a ribbon filament whose reflectivity is to be measured, Q a rochon double image prism, L a lens imaging S on the ribbon, P a disappearing filament pyrometer with pyrometer lamp at l, absorbing screen at a, limiting diaphragm at d, and colored pyrometer glass filter at f. S, L, and M are supported on a rotatable mount with a vertical axis at the center of M. S' and L' are positions corresponding to S and L for the determination of the unreflected brightness of the image of S. The locations of adjusting devices and of pinhole diaphragms for alignment testing are not shown.

* Discussion of paper by J. B. Nathanson (preceding paper in this issue: J. O. S. A. **28**, 300 (1938)), presented at the Symposium on Optical Constants, sponsored by the Optical Society of America at the New York meeting, February 24–26, 1938.

[1] Wahlin and Whitney, Phys. Rev. **50**, 735 (1936).

[2] A. G. Worthing, J. O. S. A. **13**, 635 (1926).

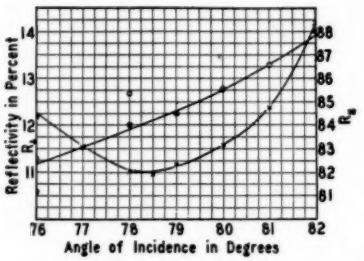

FIG. 2. Polarization produced by reflection from tungsten at room temperature for angles of incidence approximating the angle of principal incidence.

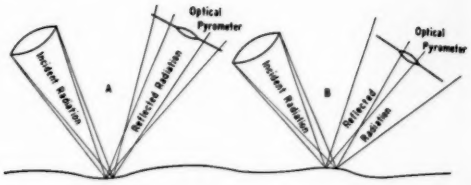

FIG. 3. Schematic drawing, greatly exaggerated, to show how (A) a small lack of planeness need not interfere with accurate measurements, and (B) too great a lack of planeness may result "as it were" in radiation holes in the beam entering the optical pyrometer.

(6) Everything considered, the optical pyrometer method, particularly if speeded up with the introduction of the photoelectric cell, merits consideration as one that has possibilities in both speed and accuracy.

Stereoscopic Drawings

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(Received April 18, 1938)

INTRODUCTION

THE existence of stereoscopic vision has been known for about 100 years. During that time, with the exception of the last few years, the use of stereographs has been generally limited to nontechnical photographic work in which a high degree of accuracy of image formation has not been a vital factor. In the few technical fields in which stereographs have been used, notably aerial photography, the emphasis has been on calculating the space scene from the two halves of the stereograph, either without reference to the stereoscopic image itself, or by the introduction of calculated pointers whose separation could be made to correspond with that of some point in the stereograph and thus measure the depth of this point without reference to the accuracy of the stereoscopic image as a whole. Thus, until recently, no great attention has been given to obtaining stereoscopic images which are faithful reproductions in shape, size and location of the original space scene.

The history of stereoscopic drawings has followed the same pattern. Such drawings have generally been used merely as illustrative material giving a clearer conception of some three-dimensional object than the usual flat illustration gives. Thus it has not been necessary to stress accuracy of image formation.

The method of attack has almost always been to develop two perspectives using the interocular distance as the separation between station points and an arbitrarily chosen picture plane. Though this leads to fairly satisfactory results it fails to achieve any great degree of accuracy because it neglects many factors involved in stereoscopic vision that are not present in simple, monocular vision.

Chief among these is the fact that every stereograph, if it is to have a fair degree of accuracy, must be viewed through a stereoscope. The optical characteristics of this stereoscope are an inherent part of the perspective system and must be considered in making the drawing. Secondly, the visual fusion of two dissimilar plane images introduces psychological problems of image interpretation which impose limitations and demand special procedures never involved in single perspective work.

It is consequently felt that a much higher degree of accuracy can be obtained in stereoscopic drawings than has been obtained. Such an increase should greatly extend the fields in which such drawings could be useful.

The present article is an attempt to develop a drawing system, together with practical details, which emphasizes accuracy of image formation. A perfect stereograph would be one in which the resultant fused image appeared to be an exact duplicate of the original space scene in shape, size and location. An attempt is made to approach this ideal as closely as possible by taking into account all the known factors entering into the formation of stereoscopic images.

GENERAL THEORY

No stereograph should be drawn for viewing without the aid of a stereoscope. Stereographs so viewed never produce accurate images. The reasons are unquestionably bound up with the breakdown of the accommodation-convergence ratio normal to binocular vision, a breakdown particularly violent in this type of viewing.

The fact of this inaccuracy can quickly be demonstrated by drawing a cube for such viewing with one face frontal. Such a cube when viewed from the calculated station points invariably appears to be shortened in depth with the receding edges not parallel. Furthermore the extent of the error seems to differ greatly with different observers. Consequently this form of viewing is given no consideration in this article.

Since a viewing instrument should be used, and since stereoscopic images are conditioned by the optical characteristics of the particular type of instrument chosen, it is necessary that each drawing be made for the type of stereoscope in which it is to be viewed. There are two chief

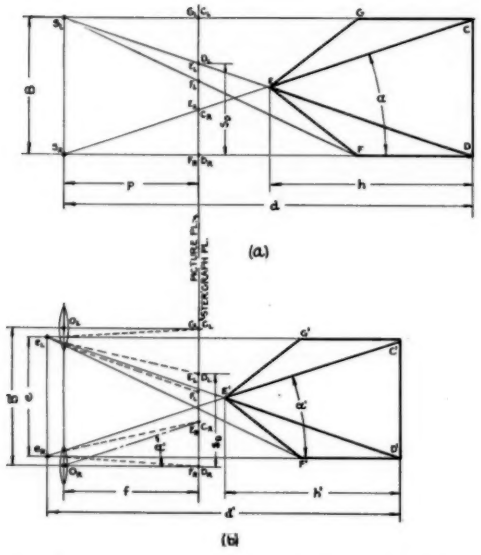

FIG. 1. Lens stereoscope. (*a*) Projection of plan view of object on drawing board (*b*) Formation of image in stereoscope.

C, D, E, F, G, points on plan view of object.
C', D', E', F', G', corresponding points on stereoscopic image.
C_L, C_R, etc., left and right projections of point C, etc., on the stereograph.
S_L and S_R, left and right station points.
O_L and O_R, left and right optical centers of stereoscope.
B, distance between station points.
B', distance between optical centers of stereoscope.
e, interocular distance.
p, distance from station points to picture plane.
f, focal length of lenses.
d, distance from station points to principal plane of object.
d', distance from eyes to principal plane of image.
e_L and e_R, left and right eyes in viewing position.
h and h', depth of object and image, respectively.
S_D, distance between two views of any point, D, on the stereograph.

types now in use, the lens type and the mirror type. A discussion of these will serve to indicate the procedure for any type.

The lens type is the most common. In stereoscopes of short focal length, up to 5 or 6 inches, full lenses with optical centers separated by the interocular distance are usually used. In stereoscopes of longer focal length, split-lenses with optical centers separated by a distance greater than the interocular distance are customary.

Let us consider a plan view of any object on a drawing board (Fig. 1a). The relations of five arbitrarily chosen points on such an object will serve to develop the principles involved. We shall attempt no rigid mathematical analysis for a general case as the five points chosen lead to the proper conclusions and greatly simplify the work.

The object is assumed to be centrally located with regard to the station points. The points C and D are first chosen at equal distances from the picture plane and so that CD is equal in length to the separation, B, between station points, and CD and the station points form a rectangle. Point E is then chosen so that the line CE passes through the right station point and DE through the left station point. Points F and G are then chosen that FD passes through the right station point and CG through the left. This makes FD and CG perpendicular to CD. Five such points may be chosen in or around any object. If the shape of the object is to be maintained the relations between these points must be maintained.

The double projection of these points onto the picture plane forms the stereograph which is then placed in a lens stereoscope. Viewed through the stereoscope, the two projections are fused by the eyes into a stereoscopic image as shown in Fig. 1*b*.

It is assumed that the stereograph is to be placed in the focal plane of the stereoscope. This relaxes the accommodation of the eyes, focusing them on infinity. It consequently eliminates the aid of accommodation in determining depth—at best a factor of very doubtful value. It also means that the accommodation is not consistent with the convergence. However, with accommodation for infinity the eyes can vary their convergence within limits without great strain or, more important for our purposes, without disturbing the interpretation of the image shape.

As commercial stereoscopes in general provide for placing the stereograph in the focal plane and as this permits the employment of a simplified, uniform drawing procedure it is highly advisable to adopt this procedure in preference to the theoretically more correct one of designing and using a stereoscope which would force accommodation at the image distance as determined theoretically from the convergence.

Consider only the shape of the resultant image. Since C_LC_R equals D_LD_R, the convergent rays through these points from the two eyes will intersect at equal distances from the eyes. Consequently the line $C'D'$ will be parallel to the focal plane regardless of the value of B' with respect to B. If $C'D'E'F'G'$ is to be similar to $CDEFG$ the angle $C'D'F'$ must be a right angle. Consequently $F'D'$ must be perpendicular to the

focal plane. Since the stereograph is placed in the focal plane of the stereoscope the optics of the lenses dictate that the ray from the eye to the stereoscopic image of any point shall be parallel to a line from the stereograph of the point for that eye to the optical center of the lens for that eye. Consequently since the line $D'F'e_R$ must be perpendicular to $C'D'$, D_RO_R must also be perpendicular to $C'D'$. This is possible only if D_R lies on the optical axis of the right lens. Exactly similar reasoning with regard to the line $C'G'$ shows C_L to be on the optical axis of the left lens. Consequently C_LD_R equals B'. Since it also equals B we conclude:

For the stereoscopic image to be properly shaped B' *must equal* B.

Furthermore to preserve the shape of the object the angle α must equal the angle α'. Since C_RD_R is a fixed distance in both figures and is perpendicular to S_RD_R and O_RD_R in the two figures the triangle $S_RC_RD_R$ must be congruent to the triangle $O_RC_RD_R$.

We conclude therefore:

For the stereoscopic image to be properly shaped p *must equal* f.

The two conclusions above are the necessary conditions for the stereoscopic image to be the same shape as the object drawn. They may be stated thus:

(1) The distance between station points on the drawing board must equal the distance between optical centers of the stereoscope.

(2) The distance from the station points to the picture plane on the drawing board must equal the focal length of the stereoscope.

Let us now consider the size and location of the image. Since the triangle ECD is similar to the triangle $E'C'D'$ both triangles are mutually similar with triangles S_LS_RE and e_Le_RE'. Thus:

$$B/e=d/d'=CD/C'D'=h/h'.$$

From this relation it is seen that if the distance between optical centers is equal to the interocular distance, $B=e$, the image will appear of its proper size and at its proper distance. ($h=h'$, $CD=C'D'$, $d=d'$.)

If the distance between optical centers is greater than the interocular distance, $B>e$, as in the standard split-lens stereoscope, the image will appear properly shaped but smaller and nearer in the ratio of the interocular distance to the optical base. Theoretically this loss of size and distance could be avoided by drawing the object proportionally larger and farther away than the image desired. Practically, since the judgment of size is largely psychological, this is usually of no importance.

Since many efforts at drawing stereographs have been based upon the calculated separation between the two views of points it is of interest to develop formulae for determining this separation. Considerations of similar triangles lead to the following:

$$S=B-pB/d \quad \text{on the drawing board,}$$
$$S=B'-fe/d' \quad \text{in the stereoscope,}$$

where S represents the distance between the two views of any point on the stereograph. The latter formula is the fundamental formula for point separation in all lens stereoscopes.

Figure 2 represents the usual type of double-reflecting mirror stereoscope. The development of the stereoscopic image of a single point on the stereograph is shown. It should be noted that the stereograph as obtained from the drawing board (Fig. 1*a*) must be cut apart and the separation

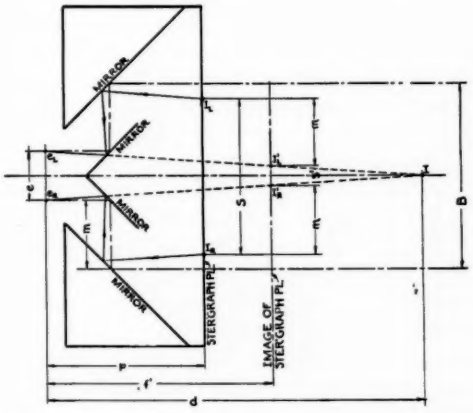

FIG. 2. Double reflecting mirror stereoscope. Formation of the image of a point.

I = stereoscopic image of point.
I_L and I_R = left and right views of point on stereograph.
I_L' and I_R' = left and right views of point on image of stereograph as seen through stereoscope.
f' = distance from eyes to image of plane of stereograph.
m = distance between mirrors taken parallel to plane of stereograph.
S = distance I_LI_R.
S' = distance $I_L'I_R'$.
p = distance of eyes from plane of stereograph.
e = interocular distance.
d = distance from eyes to stereoscopic image of point.

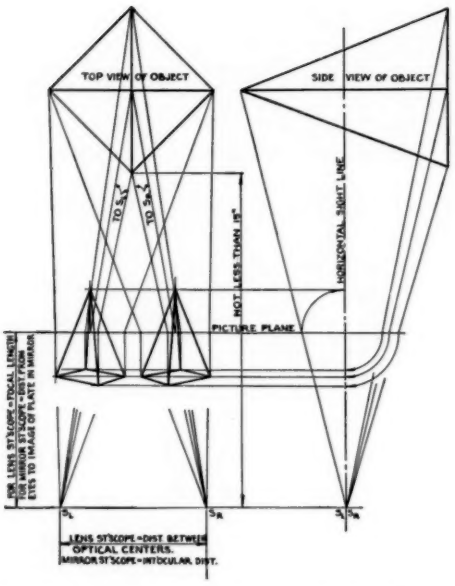
TOP VIEW OF OBJECT
SIDE VIEW OF OBJECT
HORIZONTAL SIGHT LINE
NOT LESS THAN 15"
PICTURE PLANE
LENS ST'SCOPE = DIST BETWEEN OPTICAL CENTERS.
MIRROR ST'SCOPE = INTOCULAR DIST.

FIG. 3. Development of stereograph on the drawing board.

between views increased for use in such stereoscopes.

From considerations of the formation of images in plane mirrors it will be seen that $f'=p+m$. It will be noticed that the relation of the image of the point in space, the image of the plane of the stereograph and the position of the eyes is exactly similar to that existing on the drawing board (Fig. 1*a*) among any similarly located point (in this case the point E), the picture plane and the station points. If the image is to reproduce exactly the object, this similarity must be converted into congruency; that is, if the image of the point in the stereoscope is to be made to coincide with the place of the point on the drawing board, the eyes must be made to coincide with the station points and the image of the plane of the stereograph with the picture plane. This will be achieved if e equals B and if f' equals p.

Remembering that $f'=p+m$ and observing that the separation between the two views must be increased by $2m$, we may state these conclusions:

If exact reproduction is desired in drawing for a mirror stereoscope:

(1) The distance between station points must equal the interocular distance.

(2) The distance from the station points to the picture plane must equal the distance from the eyes to the stereograph plus the distance between mirrors taken in a direction parallel to the plane of the stereograph.

(3) The separation between the two halves of the stereograph should be increased by a distance equal to twice the distance between mirrors taken in a direction parallel to the plane of the stereograph.

The formula for this type of stereoscope may be developed from similar triangles:

$$S=(e+2m)-f'e/d.$$

It should be noted that this formula corresponds to that for a lens stereoscope in which $e+2m$ corresponds to the distance between optical centers and f' to the focal length.

Figure 3 represents the system for incorporating the above conclusions for both types of stereoscope in the development of the drawing on the drawing board. The location of the picture plane and the separation between station points are established by the stereoscope to be used according to the conclusions reached above. The front point on the object is placed at least 15 inches from the station points as experience has shown that many individuals have difficulty fusing the stereographs of objects placed closer.

Two views of the object are required, a plan view for obtaining the horizontal separation between the two projections of each point, and a side view for obtaining the elevation of the projections of each point. Once these views are drawn the development of the stereograph can be done with a great deal of rapidity.

Lines are drawn from each point in the top view to the two station points. At the points where these lines intersect the picture plane, lines are drawn perpendicular to the picture plane. The stereographs of the point must lie on these lines.

Lines are then drawn from each point in the side view to the single station point. A line from this station point perpendicular to the picture plane represents the horizontal sight line. The distance of the projected points from this horizontal sight line represents the elevation of these points above or below the horizontal center line of the stereograph. This distance is rotated

through 90 degrees onto the horizontal sight line. Through these points lines are drawn parallel to the picture plane. The intersections of these lines, properly matched, with the lines drawn perpendicular to the picture plane in the top view give the two views of each point on the stereograph. In other respects the figure is self-explanatory.

The above system is simple in outline. However, it is subject to a number of practical considerations and limitations which must be taken into account if good stereographs are to result. Before presenting these it is desirable to discuss the theoretical limitations of the basic ideas involved.

Limitations of Theory of Convergence

All the foregoing is based on geometrical principles of convergence; that is, it is assumed that the stereoscopic image of any point lies at the intersection of the two projected lines from the eyes through the images of the point on the stereograph, including the effect of the stereoscope on these rays.

Though this is an acceptable assumption it should be pointed out that it is not entirely true and that therefore it does not lead to perfect results. It can be quickly proved to be untrue by cutting the two halves of the stereograph apart and varying the separation between them. It will be found that this separation may be considerably altered without any detectable effect on the image either in shape, size or location. On convergence principles the image should grow larger and recede as the halves are separated.

The question of the value of a system based on convergence principles immediately arises. This value lies in the fact that in such a system the relations between points remain consistent within the image. Thus the above system would be of little value in locating the image of a single point in space. However, given an object containing a number of points such as the pyramid in Fig. 3, the relations between these points will be consistent with the relative convergence between them for the distance to the image as a whole.

Thus the change in convergence of any point, P, on the object from the convergence of a point at the center of the object, taken to represent the position of the object as a whole, is a fixed angle, A. Whatever the distance of the image center from the eyes (experiments seem to indicate that it is almost always closer than it should be) the point P will be located correctly in the image for the indicated change in convergence, A, *for that distance.* This statement seems to be true.

A fixed ~~distance~~ difference in convergence angle between two points represents a different depth distance between these points for different values of their absolute convergence angles, that is, for different positions of the image as a whole. Consequently the distortions to be expected could be analyzed on the basis of the variation of the position of the image as a whole with respect to its correct convergent position. As we cannot accurately locate the image as a whole such an analysis will not be undertaken here. The important point for our purposes is that such distortion does exist.

Thus, though the general form remains correct, there is a change in image shape that detracts from its accuracy. However, experience has shown that the error in shape is of negligible importance unless extremely accurate judgment of line relationships is required—such as parallelism and perpendicularity within the image.

We are forced to conclude that the development of stereographs on convergence principles leads to the most accurate results that we can at present attain. The image shape only closely approximates the true shape if a stereoscope is used. The resultant distance to the image and

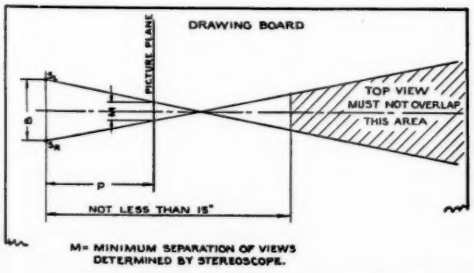

Fig. 4. Determination of limiting area for location of object.

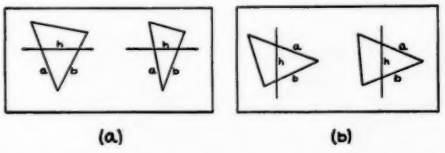

Fig. 5. Orientation of object. Avoidance of horizontal lines. (a) Poorly oriented; (b) properly oriented.

its size are also in error by an undetermined amount. However, the judgment of distance and size in such images baffles the most competent observer unless some object of known size is in the scene. In such a case the known size tends to determine the location regardless of convergence.

We shall therefore accept the above system as a satisfactory base and shall proceed to practical considerations for improving the results.

Scale of Drawings

The first thing to be considered is the scale upon which the drawing is to be made. Reasonably accurate stereographs may be drawn of simple geometrical figures on a normal scale if the greatest of care is taken in plotting points. However, such stereographs always have a roughness which can be eliminated by increasing the scale of the entire system and reducing the resultant stereograph accordingly.

The increase in scale is limited by the size of the drawing board available. A 4 to 1 increase has been found most satisfactory for lens stereoscopes, while a 2 to 1 generally suffices for mirror stereoscopes where larger prints are used. The resultant stereograph can then be reduced photographically to its proper size. Considering the minimum 15″ distance these scales require a drawing board about 80″ in length. The consequent increase in accuracy is about the maximum that can be obtained since a greater increase in scale places the object too far from the picture plane for accurate projection. The work also becomes very tedious because of this distance.

Location and Size of Object

Having chosen a suitable scale it is necessary to determine the size of the object and its distance from the station points on the drawing board. The limiting factors here are: the maximum width of the object determined from the size of the plate for the stereoscope to be used, the

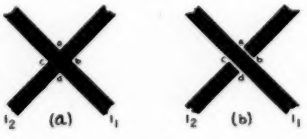

Fig. 6. Treatment of apparent intersections. (a) Improper; (b) proper.

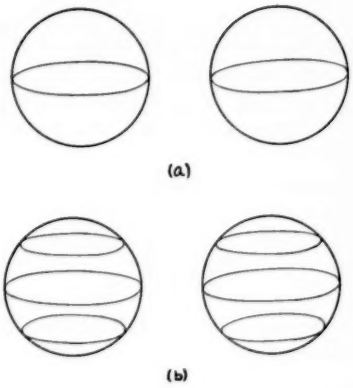

Fig. 7. Indication of curved surfaces. (a) Inadequate; (b) adequate.

minimum separation between views determined by the type of stereoscope, and the size of the drawing board. The width of the object must be such that neither projection on the picture plane exceeds the maximum allowable width of view visible in the stereoscope. The same restriction applies to the height. The maximum depth of the object is in general determined by the size of the drawing board. Though theoretically it is possible to extend the depth an infinite amount it must be remembered that a receding line should grow thinner. The edges of objects of excessive depth tend to assume the proportions of a cable if a uniform line thickness is used. This restriction, however, does not present itself unless distant objects are attempted.

Assuming no overlapping is to be allowed, lines may be drawn on the drawing board as shown in Fig. 4 for determining the limiting size of the object. Since separation of the two views causes no change in image shape, overlapping with arbitrary separation afterwards is an allowable practice. This permits of somewhat larger sizes for near objects.

As has been stated, the forward edge of the object should not be closer than 15″ from the station points. In addition to the fact that some individuals have difficulty fusing the stereographs of closer objects, such objects make the change in convergence from front to back of the object, if it has any considerable depth, so great that the doubling of the portion not converged upon becomes an annoying feature. In particular

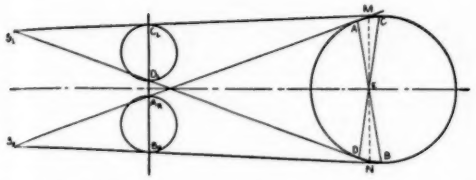

FIG. 8. Determination of the contour circles of a sphere.

the eyes will be unable to fuse completely the entire length of a receding line.

ORIENTATION OF THE OBJECT

It must be remembered that in all stereographs the line of sight is perpendicular to the plane of the stereograph and aimed approximately at its mid-point. Thus the proper viewing point is fixed in advance and cannot be chosen by the draftsman. This fact means the horizon must be taken close to, or through, the center of the object. Thus the ordinary perspective method of considering a principal plane of the object as horizontal and raising or lowering the horizon in order to get a view from above or below is not feasible in drawing stereographs.

Consequently, if a view is desired from above or below it must be obtained by tipping the object. Such a procedure is usually desirable since most objects normally contain horizontal planes which closely approach an edge view with respect to an observer looking at their midpoint. In such a view the accuracy with which details on the plane must be drawn increases enormously as the plane approaches an edge. Here a small vertical error on the stereograph becomes a very large depth error on the surface of the stereoscopic image of the plane. Consequently it is desirable that the position of the object be so chosen that no plane containing any details approaches an edge view. Experience has led to the arbitrary rule that normally horizontal planes should make an angle with the horizontal sight line whose tangent is not less than 0.25.

This restriction, so far as it is possible to comply with it, also applies to any plane, horizontal or otherwise, that may approach an edge view. As a corollary the extension of no plane in the object, vertical or otherwise, should intersect the line segment between the station points. Such planes are seen from one side by one eye and from the other side by the other eye and are almost edge views to both. This leads to difficulty in original fusion and then to confusion of interpretation because different sides of the same plane are seen. Further comments on this point will be found in the discussion of visibility.

Horizontal lines, i.e., any line in the plane of the horizon or any horizontal-frontal line, should be avoided unless the line is short and both ends are tied to other lines. A straight, uniform, horizontal line presents no angle of convergence except at the end points. Consequently its relative convergence with respect to the rest of the object cannot be determined unless the ends are fixed upon.

If such a line does appear it has the tendency to float in the stereograph and to change in depth along the line in accordance with the depth of lines which it appears to cross.

In Fig. 5a, if the line *a*, the rear line, is fixed upon, the line *h* will tend to appear at the depth of *a* at the apparent point of intersection. If *b*, the forward line, is fixed upon, *h* will tend to appear at the depth of *b* at the apparent point of intersection. Thus *h* will appear to come forward as the eyes move from *a* to *b*. That there is nothing to prevent this occurrence can best be seen by remembering that the stereograph of any horizontal-frontal line is exactly the same as the stereograph of a plane containing that line and the line segment connecting the station points. Thus the eyes are free to interpret such a line as the edge view of this plane and consequently may interpret any point on this line at any depth whatever in the plane.

This effect can best be avoided by choosing some other orientation in space for the entire figure. Fig. 5b would solve the problem for this particular example.

It is, of course, not always possible to orient the object in such a way as to eliminate all planes approaching an edge view and all horizontal lines. However, many of the worst situations can usually be avoided.

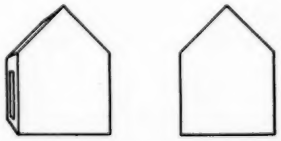

FIG. 9. Improper orientation of receding planes. The left side of the house is visible in only one view.

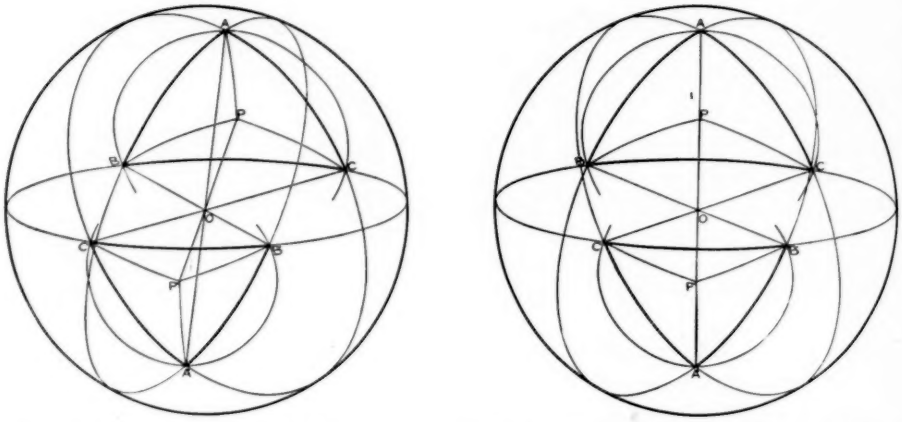

FIG. 10. Solid geometry. For use in proving theorems involving symmetric triangles. Published by permission of the Keystone View Company.

TREATMENT OF INTERSECTIONS

The tendency for crossing lines to appear as actually intersecting in space is not fully eliminated by the avoidance of horizontal lines. This is particularly true if the drawings are inked drawings in which the lines must necessarily have some width. The fusion of the ink at the point of apparent intersection eliminates the edges of the forward line. Such edges always exist in space vision. Thus if the eyes concentrate their attention at this point the true separation of the lines is liable to be lost. Fig. 6a represents a magnified intersection of lines as inked on a stereograph. Suppose l_1 is the forward line in space. Its edges, *ab* and *cd*, would be visible in space but they do not appear on the stereograph. Small as this fault appears, it causes a very annoying confusion in viewing the stereograph, particularly if one of the lines approaches the horizontal.

This confusion can best be eliminated in accurate work by using a razor blade to make breaks in l_2, the rear line, where it passes behind l_1, as shown in Fig. 6b. Every intersection which is not a true intersection in space should be treated in this manner on the finished stereograph. These breaks on the originals and on photographic reproductions of Figs. 10 and 11 are too thin to show in a printed reproduction.

CURVED SURFACES

In dealing with plane surfaces, if two or more lines are shown in any plane the psychological tendency is to read the plane into the picture. Thus in Fig. 3 only three lines are shown in each face of the pyramid yet the mind interprets them as being plane faces, not as merely triangles of wire.

On a curved surface the mind does not readily interpolate the proper surfaces into the empty spaces. Thus the sphere shown in Fig. 7a would not be interpreted as a sphere but as two circular loops of wire with open spaces between. Consequently enough lines must be shown to permit the mind to interpolate the intervening surfaces in the proper manner.

Closed surfaces of uniform curvature, either single or double, such as cylinders or spheres, can usually be shown with a minimum of four properly chosen loops. Fig. 7b approaches this but would be further improved if another circle were added in a vertical plane.

With surfaces of changing curvature the number of lines required depends upon the complexity of the surface. Experience is the only guide here.

Difficulty is encountered on the edges of curved surfaces as no such edges exist in reality. Consider the sphere in Fig. 8. The circle, *AB*, not a great circle, projects on the stereograph as a limiting ellipse in the right view. The circle *CD* projects as the limiting ellipse in the left view. These two ellipses are fused in the stereoscopic image though they are not projections of the same circle; such fusion does not produce a true image of the sphere. Their fused image

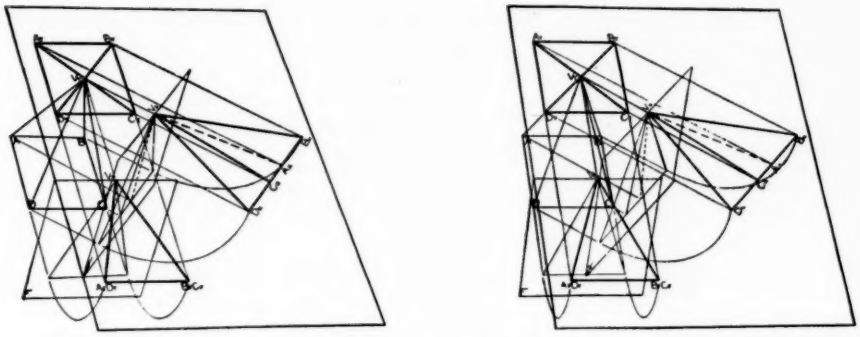

FIG. 11. Descriptive geometry. Showing the method of developing an auxiliary elevation.

actually represents the ellipse tangent to the sphere at the two points represented by E and having MN as a major axis.

An accurate mathematical solution of this problem cannot be given since the psychology of the interpretation of a sphere in binocular vision is not clearly understood. As a practical matter the drafting process is greatly simplified if such limiting ellipses are treated as circles of the diameter, AB. These ellipses fuse as a circle having MN as a diameter, slightly off the surface of the sphere and forward from the center. In the resultant image this circle tends to be interpreted as a great circle so that more of the sphere seems to lie behind it than before, giving the impression of an object slightly egg-shaped toward the rear.

In practice it has been found satisfactory to use thickened lines for this circle. Apparently they produce a vagueness of fusion which somewhat approximates what happens when a real sphere is viewed in space.

All other circles must be plotted as ellipses. An enclosing square divided equally in one direction and harmonically in the other forms a convenient gridwork for such plotting.

Many similar difficulties present themselves in the drawing of curved surfaces. The best that can be said is that these must be worked out empirically as they arise. Many can be avoided by a proper orientation of the object.

VISIBILITY

One of the great values of a stereograph is that in it simple geometrical figures may be treated as though they were transparent. In such cases the problem of visibility does not arise since the depth of each line is obvious. The ordinary method of showing hidden lines by short dashes is in general not satisfactory, for the illusion of reality in a stereograph destroys the acceptance of the convention as valid, the lines looking like a series of unattached wires. Furthermore such dashes must be coordinated in the two views, a very laborious process.

If it is desirable to distinguish between lines on front surfaces and those behind some surface this can best be done by using lines of distinctly different weight.

If it is desired to consider the object as opaque, in the stereograph of a house, for instance, the invisible lines can be omitted entirely. Care should be taken that no lines are visible in one view and invisible in the other. This situation occurs on receding planes which pass between the eyes. Thus the left side of the house in Fig. 9 is visible to the left eye but invisible to the right. Consequently the window shown will appear in only one view.

Because a stereograph lacks the color and shading present in ordinary binocular vision the window "floats" in space or, to persons with a weak left eye, is entirely invisible. This difficulty can be avoided by placing the house in such a position that no principal plane in it intersects the line segment between the station points.

LETTERING

If lettering is to be done it is essential that both halves of the stereograph be lettered. Lettering placed on only one half is not fixed in space and consequently "floats." Furthermore individuals with a strong master eye fail to see such lettering at all if it is on the side of the weak eye.

It is possible to place letters on the top and

side views and project the points in the same manner as the rest of the drawing is projected. This is extremely tedious. It is much more convenient to construct the letters directly on the finished stereograph.

Whenever possible, letters should be placed within some plane of the object. Letters placed in frontal planes, independent of the object, tend to float even though they should be fixed. Furthermore, such letters detract from the sense of reality by being hung in space with no visible means of support.

The most satisfactory system is to construct squares or rectangles in the planes of the finished stereograph and to draw the letters in them freehand. All the squares may first be constructed in one view and rapidly found in the other view by projecting horizontally the intersections of the prolonged edges with some line in the object to the corresponding point on the other view. To do this, one should place the edges of the square as close as possible to the edges of the object. The edges of the square may be drawn to the vanishing point of a parallel edge of the object if the vanishing point is available. If it is not available the error in drawing them parallel to the nearest edge will be negligible.

It is well to place letters in the plane most nearly frontal where a choice is possible, as on receding planes the widths of the letters either beome so small as to make it difficult to construct them with accuracy or they are greatly out of proportion in having one dimension excessive with respect to the other.

Guide lines for letters on curved surfaces may be drawn parallel to a line tangent to an adjacent curve at the point nearest the letter or the adjacent curve may be duplicated for short distances.

Shading

Shading may be done if great care is taken to cover the same area in both views and to avoid lines or accidental patterns of any sort. The use of lines necessitates matching them in the two views so that their fused images will lie in the proper surface. This is practically a prohibitive procedure.

In using a wash for surface shading, care must be taken that uniformity is obtained. Accidental dark or light spots on the two views tend to fuse, thus establishing a convergence angle which draws the shading out of its proper plane.

Washes which completely cover surfaces bounded by lines can give very satisfactory results if carefully applied.

Choice of Paper

The surface on which the stereograph is to be made should be carefully chosen. The presence of texture in this surface establishes a definite plane in the field of view in the stereoscope. The visibility of this plane is unquestionably one of the factors which causes distortion in stereoscopic images, tending to draw them forward and decrease their relative depth. The more prominent this plane, the greater the distortion.

Consequently hard surface, glossy paper should be chosen for stereoscopic drawings. The reproduction of such drawings negatively, that is, white lines on a black background is a highly satisfactory method of minimizing the visibility of this plane of the stereograph.

Conclusion

The above discussion has developed a system of drawing stereographs based primarily on convergence principles. The draftsman should constantly remember, however, that once such a system has been devised his job has just begun, for the formation and interpretation of stereoscopic images are not reducible to a completely rigid, mathematical system. Though each interpretation is made instantly by the eyes and brain, many minute details condition this interpretation and frequently contradict the convergent evidence.

Many of these details that lead to confusion or erroneous interpretation have been pointed out and methods devised for avoiding them. It is attention to such details that leads to accurate stereographs. As the degree of accuracy increases the number of fields in which stereoscopic drawings may be of value increases. This fact should in turn lead to a more exacting demand for such accuracy and consequently to a more thorough investigation of the field.